Aircraft conflict resolution and recovery with mixed integer optimization

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Outline

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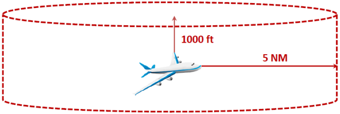
Air Traffic Control

Air Conflict

Separation standards (1000 ft $\approx$ 300m - 5 NM $\approx$ 10km):

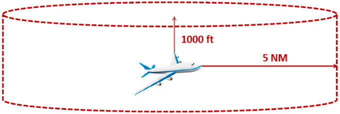


Two aircraft are in **conflict** if they violate the separation standards

Air Traffic Control

Air Conflict

Separation standards (1000 ft ≈ 300m - 5 NM ≈ 10km):



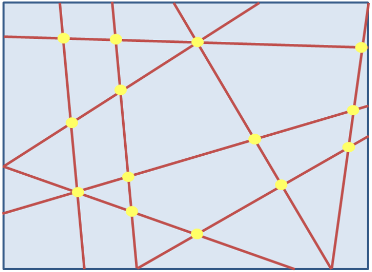
Two aircraft are in **conflict** if they violate the separation standards

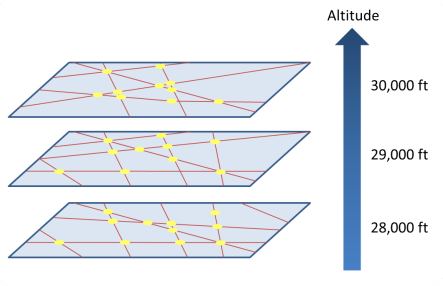
Cognitive Workload

- Controllers' workload is conditioned by the detection and the resolution of air conflicts
- If controllers' cognitive capacity is saturated → flight safety is at risk, delay increases

En-route Air Traffic Networks

En-route Air Traffic Networks are designed into linear flight trajectories and separated flights level. At each flight level, flight trajectories intersect at waypoints (geo-referenced position).





→ Focus on 2D problem

Outline

Literature Review

Gaps in the literature:

- Very little attention to models with aircraft trajectory recovery
- Conflict Resolution with Recovery is very difficult to solve and a typical approach is to discretised the solution space which is somehow limiting.
- Coordinate manoeuvres: aircraft have synchronous movement as proposed;
- Speed control as the only manoeuvres is a valid solution, however might not be enough to solve large instance with dense traffic scenarios;

Aircraft Separation Modelling

Let $\boldsymbol{p}_i(t) = [x_i(t), y_i(t)]^\intercal$ be the vector representing the position of flight i at time t. The relative position of aircraft i and j at time t can be represented as $\boldsymbol{p}_{ij}(t) = \boldsymbol{p}_i(t) - \boldsymbol{p}_j(t)$. Let d be the horizontal separation norm, the two aircraft are separated if and only if:

$$\|\boldsymbol{p}_{ij}(t)\| \geqslant d, \forall t \geqslant 0 \tag{1}$$

Let $\boldsymbol{v}_{ij} = [v_{ij,x}, v_{ij,y}]^\intercal$ be the relative velocity vector of i and j, *i.e.* $v_{ij,x} = v_{i,x} - v_{j,x}$ and $v_{ij,y} = v_{i,y} - v_{j,y}$, and let $\widehat{\boldsymbol{p}}_{ij} = [\widehat{x}_{ij}, \widehat{y}_{ij}]^\intercal$ be their relative initial positions. Assuming that uniform motion laws apply, $\boldsymbol{p}_{ij}(t)$ can be expressed as: $\boldsymbol{p}_{ij}(t) = \widehat{\boldsymbol{p}}_{ij} + \boldsymbol{v}_{ij}t$.

Aircraft Separation Modelling

For each aircraft $i \in A$:

- initial speed $\widehat{v}_i$, initial heading $\widehat{\theta}_i$
- θ_j be the heading deviation angle ($\theta_i = 0$ means no deviation)
- q_i be the speed control variation ($q_i = 1$ means no variation in speed)

Aircraft velocity components are $v_{i,x} = q_i \widehat{v}_i \cos(\theta_i + \widehat{\theta}_i)$ and $v_{i,y} = q_i \widehat{v}_i \sin(\theta_i + \widehat{\theta}_i)$. Aircraft relative velocity vector components:

$$v_{ij,x} = q_i \widehat{v}_i \cos(\theta_i + \widehat{\theta}_i) - q_j \widehat{v}_j \cos(\theta_j + \widehat{\theta}_j) \quad (2)$$

$$v_{ij,y} = q_i \widehat{v}_i \sin(\theta_i + \widehat{\theta}_i) - q_j \widehat{v}_j \sin(\theta_j + \widehat{\theta}_j) \quad (3)$$

Aircraft Separation Modelling

Approach → determine the time of minimum separation based on aircraft relative motion. Squaring Eq. (1):

$$f_{ij}(t) \equiv \|\boldsymbol{v}_{ij}\|^2 t^2 + 2\widehat{\boldsymbol{p}}_{ij} \cdot \boldsymbol{v}_{ij} t + \|\widehat{\boldsymbol{p}}_{ij}\|^2 - d^2 \geqslant 0 \quad (4)$$

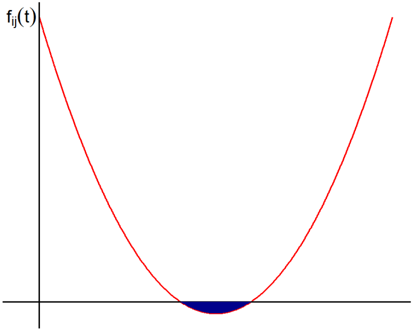
$f_{ij}(t)$ is a 2nd order convex polynomial in t. Let t_{ij}^m be the time at which $f_{ij}(t)$ is minimal

$$f'_{ij}(t) = 0 \quad \Leftrightarrow \quad t_{ij}^m = \frac{-\widehat{\boldsymbol{p}}_{ij} \cdot \boldsymbol{v}_{ij}}{\|\boldsymbol{v}_{ij}\|^2} \quad (5)$$

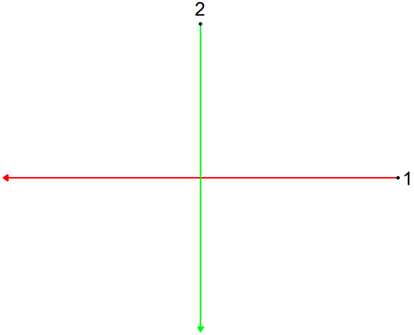
The sign of the inner product $\widehat{\boldsymbol{p}}_{ij} \cdot \boldsymbol{v}_{ij}$ indicates aircraft convergence/divergence.

Substituting t_{ij}^m in $f_{ij}(t)$, the separation condition (4) can be simplified to $f_{ij}(t_{ij}^m) \geqslant 0$, which does not depend on t anymore.

Conflict Avoidance

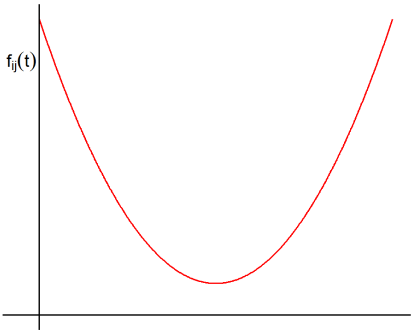


(a) Conflict region highlighted in blue

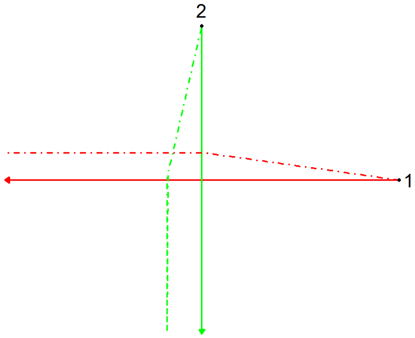


(b) Conflicting trajectories

Conflict Avoidance



(c) $f_{ij}(t)$



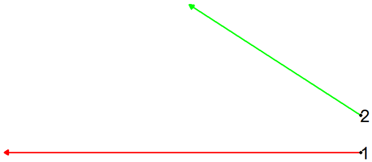
(d) Conflic-free trajectories

Separation Configuration

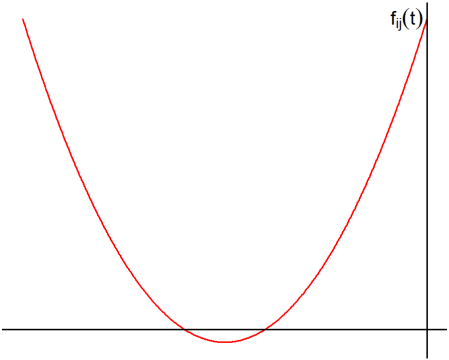
Therefore, each pair must be checked for the configuration:

- Divergence → no conflict between aircrafts;
- Convergence before turning times → aircrafts in convergence trajectory and the time of minimum separation occurs before one of the aircraft turns;
- Convergence after turning times → comparison between turning time and time of separation loss;

Separation Configuration

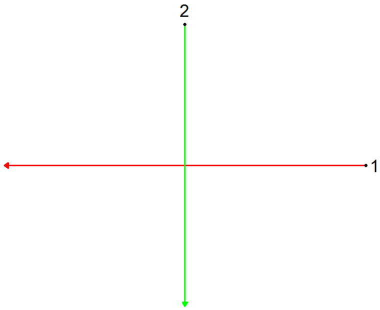


(a) Divergence trajectories

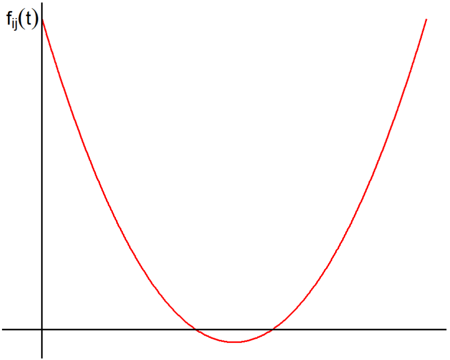


(b) Conflict point is on the past

Separation Configuration

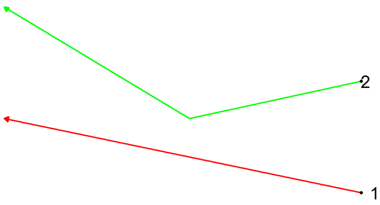


(a) Convergence trajectories

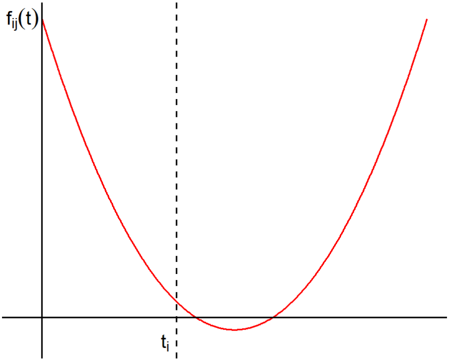


(b) Conflict point is on the future and separation must be enforced

Separation Configuration



(a) Convergence trajectories



(b) Conflict point is on the future but one of aircraft turns first

Aircraft Separation Modelling

During recovery, for each aircraft $i \in A$:

- initial speed $\widehat{v}_i$, initial heading $\widehat{\theta}_i$
- t_i be the recovery time where heading and speed are recovered;

Segment Comparison

For each pair of aircraft, we need to compared the following segment of trajectories:

- A_i, A_j: both aircraft in action path;
- A_i, R_j or R_i, A_j: one aircraft in action another in recovery;
- R_i, R_j: both aircraft in recovery path;

Motion equations

During action:

$$x_i = \hat{x} + q_i \widehat{v}_i \cos(\theta_i + \widehat{\theta}_i) t \tag{6}$$

$$y_i = \hat{y} + q_i \widehat{v}_i \sin(\theta_i + \widehat{\theta}_i) t \tag{7}$$

During recovery:

$$x_i = \hat{x} + q_i \widehat{v}_i \cos(\theta_i + \widehat{\theta}_i) t_i + \widehat{v}_i \cos(\widehat{\theta}_i)(t - t_i) \tag{8}$$

$$y_i = \hat{y} + q_i \widehat{v}_i \sin(\theta_i + \widehat{\theta}_i) t_i + \widehat{v}_i \sin(\widehat{\theta}_i)(t - t_i) \tag{9}$$

Initial Approach

Full Action and Recovery Model:

- Optimization of speed control, heading angle and turning time for each aircraft providing a parallel conflict free trajectory;
- Non-linear and complexity caused by trigonometric operator involving decision variables in constraints;
- Alternative: 2-step decomposition;

Outline

Approach

2 steps Iterative Action and Recovery:

- Action Model → speed control and heading changes are optimized;
- Recovery → recovery (or turning) time;
- Solution from recovery is accounted in the next iteration of action model through an iterative process.

Approach

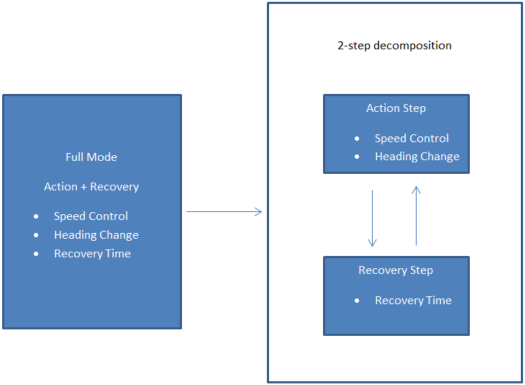


Figure: Iterative process

Action Model

- Minimize deviation considering speed control and heading changes as the only manoeuvres;
- The objective is to find the optimal combination of speed changes and heading angles turn to avoid any conflict between all pair of aircraft;
- Initial position are fixed and independent of any variables.

Action Step

Model (Action Model)

$$\text{min} \quad \sum_{i \in \mathcal{A}} (\theta_i)^2 + \delta(1 - q_i)^2$$

subject to

$$v_{ij,x} = \hat{v}_i q_i \cos(\hat{\theta}_i + \theta_i) - \hat{v}_j q_j \cos(\hat{\theta}_j + \theta_j) \qquad \forall (i,j) \in \mathcal{P}$$

$$v_{ij,y} = \hat{v}_i q_i \sin(\hat{\theta}_i + \theta_i) - \hat{v}_j q_j \sin(\hat{\theta}_j + \theta_j) \qquad \forall (i,j) \in \mathcal{P}$$

$$v_{ij,x}^2(\hat{y}_{ij}^2 - d^2) + v_{ij,y}^2(\hat{x}_{ij}^2 - d^2) - v_{ij,x}v_{ij,y}(2\hat{x}_{ij}\hat{y}_{ij}) \geqslant (z_{ij}^{sep} - 1)M \qquad \forall (i,j) \in \mathcal{P}$$

$$-(v_{ij,x}\hat{x}_{ij} + v_{ij,y}\hat{y}_{ij}) \leqslant (1 - z_{ij}^{div})M \qquad \forall (i,j) \in \mathcal{P}$$

$$z_{ij}^{div} + z_{ij}^{sep} \geqslant 1 \qquad \forall (i,j) \in \mathcal{P}$$

$$\underline{q} \leqslant q \leqslant \bar{q} \qquad \forall i \in \mathcal{A}$$

$$\underline{\theta} \leqslant \theta \leqslant \bar{\theta} \qquad \forall i \in \mathcal{A}$$

$$v_{ij,x}, v_{ij,y} \in \mathbb{R} \qquad \forall (i,j) \in \mathcal{P}$$

$$z_{ij}^{div}, z_{ij}^{sep} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{P}$$

δ is a preference weight to balance heading and speed control, M is a large positive number

Parallel Recovery Model

- Minimize deviation considering heading changes as the only manoeuvre;
- The objective is to find the optimal time where each aircraft can return to a parallel trajectory performing the same heading manoeuvres an in the action;
- Parallel heading recovery and speed recovery;
- Initial position are parametrized as function of turning time.

Recovery Step

$$
\begin{array}{ll}
\textit{minimize} \quad \sum_{i \in \mathcal{A}} \frac{t_i^2 q_i^2 \hat{v}_i^2 \sin(2\theta)}{4} & \\
\textit{subject to} & \\
t_i \leq t_j - (1-\gamma_{ij})M & \forall (i,j) \in \mathcal{P} \\
t_j \leq t_i - \gamma_{ij} M & \forall (i,j) \in \mathcal{P} \\
z_{ij}^{sep1} + z_{ij}^{div1} + z_{ij}^{conv1} \geq (1-\gamma_{ij}) & \forall (i,j) \in \mathcal{P}_{\mathcal{AR}} \\
A_{ij} t_j^2 + B_{ij} t_j + C_{ij} \geq M(z_{ij}^{sep1} - 1) & \forall (i,j) \in \mathcal{P}_{\mathcal{AR}} \\
-t_j(2D_{ij} + E_{ij}) - F_{ij} \leq (1 - z_{ij}^{div1})M & \forall (i,j) \in \mathcal{P}_{\mathcal{AR}} \\
t_i - G_{ij} \leq (1 - z_{ij}^{conv1})M & \forall (i,j) \in \mathcal{P}_{\mathcal{AR}} \\
z_{ij}^{sep2} + z_{ij}^{div2} + z_{ij}^{conv2} \geq \gamma_{ij} & \forall (i,j) \in \mathcal{P}_{\mathcal{RA}} \\
H_{ij} t_j^2 + I_{ij} t_j + J_{ij} \geq M(z_{ij}^{sep2} - 1) & \forall (i,j) \in \mathcal{P}_{\mathcal{RA}} \\
-t_j(2K_{ij} + L_{ij}) - N_{ij} \leq (1 - z_{ij}^{div2})M & \forall (i,j) \in \mathcal{P}_{\mathcal{RA}} \\
t_i - O_{ij} \leq (1 - z_{ij}^{conv2}) & \forall (i,j) \in \mathcal{P}_{\mathcal{RA}} \\
z_{ij}^{sep3} + z_{ij}^{conv3} + z_{ij}^{conv4} \geq 1 & \forall (i,j) \in \mathcal{P}_{\mathcal{RR}} \\
Q_{ij} t_i^2 + R_{ij} t_j^2 + S_{ij} t_i t_j + T_{ij} t_i + U_{ij} t_j + V_{ij} \geq M(z_{ij}^{sep3} - 1) & \forall (i,j) \in \mathcal{P}_{\mathcal{RR}} \\
t_i(-U_{ij} - 2Q_{ij}) + t_j(-V_{ij}) + (-X_{ij}) \leq (1 - z_{ij}^{conv3})M & \forall (i,j) \in \mathcal{P}_{\mathcal{RR}} \\
t_j(-V_{ij} - 2Q_{ij}) + t_i(-U_{ij}) + (-X_{ij}) \leq (1 - z_{ij}^{conv4})M & \forall (i,j) \in \mathcal{P}_{\mathcal{RR}} \\
z_{ij}^{sep1}, z_{ij}^{div1}, z_{ij}^{conv1} \in \{0,1\} & \forall (i,j) \in \mathcal{P}_{\mathcal{AR}} \\
z_{ij}^{sep2}, z_{ij}^{div2}, z_{ij}^{conv2} \in \{0,1\} & \forall (i,j) \in \mathcal{P}_{\mathcal{RA}} \\
z_{ij}^{sep3}, z_{ij}^{conv3}, z_{ij}^{conv4} \in \{0,1\} & \forall (i,j) \in \mathcal{P}_{\mathcal{RR}} \\
\gamma_{ij} \in \{0,1\} & \forall (i,j) \in \mathcal{P} \\
\underline{t_i} \leq t_i \leq \bar{t_i} & \forall i \in \mathcal{A}
\end{array}
$$

Modelling of Cost of Avoidance and Recovery

Cost of conflict avoidance and recovery is taking into account in next iteration

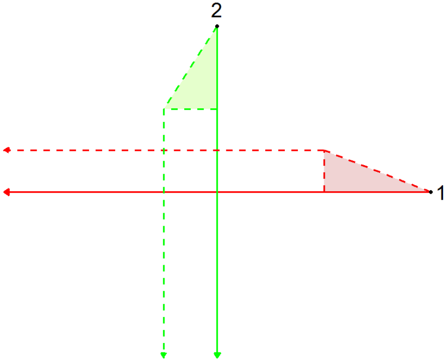


Figure: Area between nominal and optimized route represent the cost of avoidance and recovery

Outline

Results on the Circle Problem

$\|A\|$	*nc*	*Objective Function - Action*	*Objective Function - Recove*
CP4	6	0.00124988	0.284523
CP5	10	0.00227346	0.034996
CP6	15	0.00361866	0.426523
CP7	21	0.00474705	0.129595
CP8	28	0.00692085	0.568877
CP9	36	0.00862191	0.250185
CP10	45	0.0110994	0.508209

Table: Results on the benchmark instances for 2-step decomposition using CP instances

Results on the Random Circle Problem

$\|A\|$	*nc*	*Objective Function - Action*	*Objective Function - Recove*
RCP1	4	0.000426197	0.0437476
RCP2	4	6.41035e-05	0.0125
RCP3	3	0.000307855	0.01
RCP4	4	1.0026e-05	0.0324106
RCP5	3	0.000114419	0.015
RCP6	4	0.0011803	0.11415
RCP7	4	0.000306272	0.0564558
RCP8	3	0.000386562	0.0776211

Table: Results on the benchmark instances for 3-step decomposition using RCP instances

Cost of Avoidance and Recovery

Applying the iterative process:

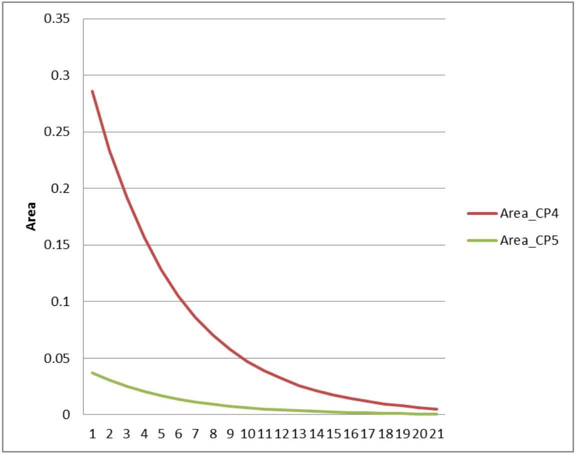
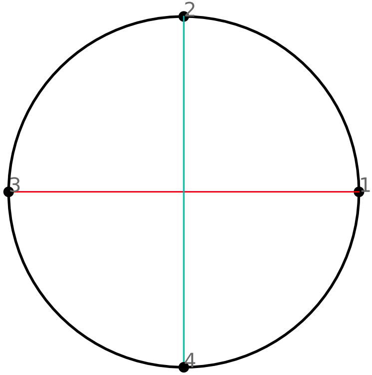


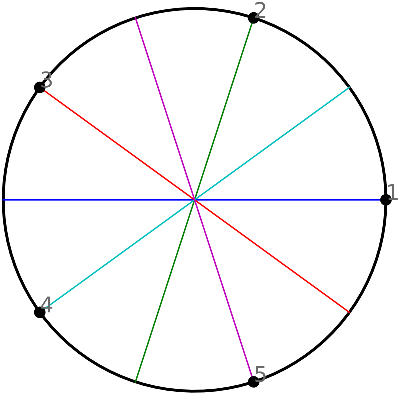
Figure: Progress of cost of avoidance and recovery throughout iterative process

Cost of Avoidance and Recovery - Trajectories

Comparison of cost throughout iterative process:

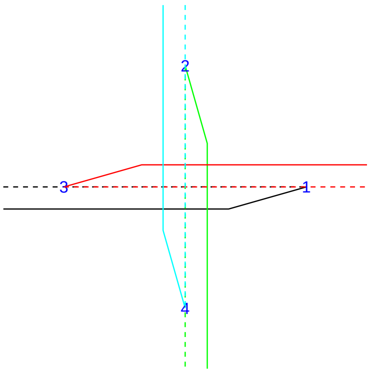


(a) Initial Solution Configuration

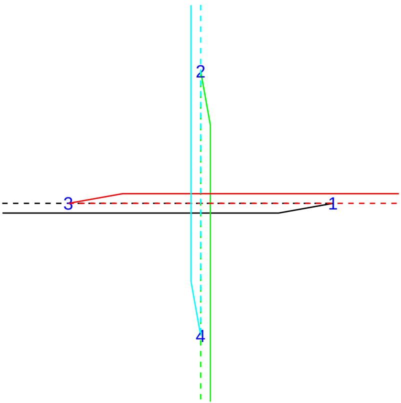


(b) After iterative process

Cost of Avoidance and Recovery - Trajectories



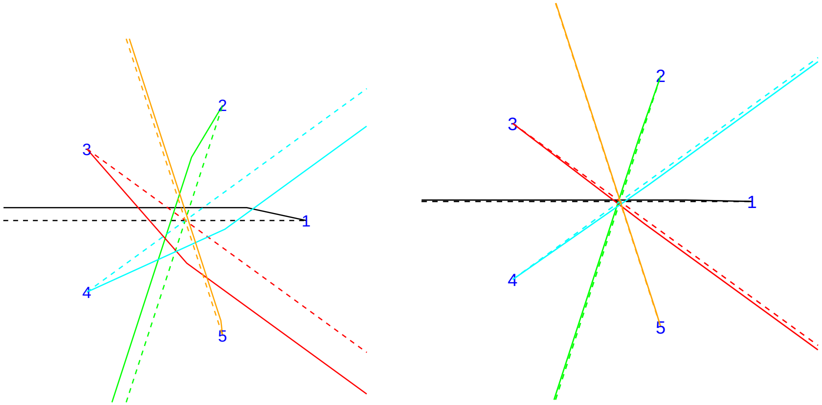
(a) Initial Configuration CP-4



(b) Initial Configuration CP-5

Cost of Avoidance and Recovery - Trajectories

Comparison of cost throughout iterative process:



(a) Initial Solution Configuration (b) After iterative process

Potential area of application

- Motion Planning;
- Autonomous Vehicles Control at Intersections;
- Drone traffic Control;

Thank you for your attention