Autonomous vehicle lanes for a seamless transition to future mobility

Shantanu Chakraborty

with

Dr. David Rey | Dr. Michael W. Levin | Prof. S. Travis Waller

TRANSW

Transport Research Association for NSW

2019

Research Center for Integrated Transport Innovation (rCITI)

School of Civil and Environmental Engineering

UNSW Sydney

Outline

Introduction

Introduction

- Autonomous vehicles (AV) may be available on the mass market by 2025, two decades after DARPA Grand Challenge's successful tests
- Australia's first on-demand driverless car was bought by RAC WA in Sept. 2018
- Interaction between legacy vehicles and AVs is inevitable in the upcoming decades

- With the introduction of AVs, System Optimal Dynamic Traffic Assignment (SODTA) will become more relevant

Research Questions

- How do we model a network with legacy and autonomous vehicles (LV & AV) with a system level objective?
- Knowing the percentage of AVs on road, how can we improve traffic infrastructure for maximum benefits?
- How the presence of AVs would affect the network performance?
- Knowing the utility/disutility of AVs and LVs, how do we design AVs as a service for a network?

Methodology

System Optimum DTA for Network Design

Develop a Link Transmission Model (LTM) based system optimum dynamic traffic assignment (SODTA) formulation

Deployment of AV lanes with Endogenous Demand

Find the optimal lane allocation for AVs with a mode choice model for endogenous demand

System Optimum DTA for Network Design

Link Transmission Model for Network Design

- LTM is a DNL model that determines time-dependent link volumes, link travel times, route travel times
- Sending flow ($S_{i,t}$): Max no. of vehicles leaving d/s end during interval $[t, t+\delta]$
- Receiving flow ($R_{j,t}$): Max no. of vehicles entering u/s end during interval $[t, t+\delta]$
- $S_{i,t}$ and $R_{j,t}$ are constrained by the traffic flow model
- Transition flow ($G_{ij,t}$): No. of vehicles transferred from link i to j during interval $[t, t+\delta]$ $\Rightarrow$ determined by node model

Yperman, I., Logghe, S. and Immers, B. (2005) The link transmission model: An efficient implementation of the kinematic wave theory in traffic networks. In Proceedings of the 10th EWGT Meeting, Poznan, Poland.

Link Transmission Model for Network Design

Objective function:

$$\min \delta \sum_{i\in A_C} \sum_{k\in A_S} \sum_{t\in T} (z^{k+}_{i,lv}(t) - z^{k-}_{i,lv}(t))$$

subject to, Demand constraint:

$$z^{k+}_{i,lv}(t) = \sum_{t'<t} d^{t'}_{i,k} \qquad \forall i \in A_R, \forall k \in A_S, \forall t \in R$$

Cumulative vehicle numbers(u/s):

$$z^{k+}_{i,lv}(t) = \sum_{t'<t} \sum_{h\in\Gamma^-(i)} y^{t'}_{h,i,k} \qquad \forall i \in A_D, \forall k \in A_S, \forall t \in T$$

Cumulative vehicle numbers(d/s):

$$z^{k-}_{i,lv}(t) = \sum_{t'<t} \sum_{j\in\Gamma^+(i)} y^{t'}_{i,j,k} \qquad \forall i \in A_C, \forall k \in A_S, \forall t \in T$$

Link Transmission Model for Network Design

Constraints on sending flow

Flow propagation constraint:

$$\sum_{j \in \Gamma^+(i)} y^k_{i,j,lv}(t) \leq \left(z^{t_s+}_{i,k} - z^{k-}_{i,lv}(t)\right) \quad \forall i \in A_C, \forall k \in A_S, \forall t \in T \backslash \{t_n\}$$

Capacity constraint:

$$\sum_{k \in A_S} \sum_{j \in \Gamma^+(i)} y^k_{i,j,lv}(t) \leq \delta q_i \qquad \forall i \in A_C, \forall t \in T$$

Link Transmission Model for Network Design

Constraints on receiving flow

Flow propagation constraint:

$$\sum_{i\in\Gamma^-(j)} \sum_{k\in A_S} y^k_{i,j,lv}(t) \leq \left(K^{jam}_j L_j - \left(z^{t+}_{j,k'} - z^{t_r-}_{j,k'}\right)\right)$$

$$\forall j \in A_D, \forall k' \in A_S, \forall t \in T$$

Capacity constraint:

$$\sum_{i\in\Gamma^-(j)} \sum_{k\in A_S} y^k_{i,j,lv}(t) \leq \delta q_j \qquad \forall j \in A_D, \forall t \in T$$

Trip completion

$$\sum_{h\in\Gamma^-(k)} z^{t_n-}_{h,k} = \sum_{i\in A_R} \sum_{t\in R\setminus t_n} d^{0,k,t}_i \qquad \forall k \in A_S$$

Summary

- An LTM based LP formulation is proposed and compared with its CTM counterpart to solve a dynamic NDP under SO traffic conditions.
- There is no incentive to allocate non-uniform budget to the cells of the same link as cell transfer flow is limited by the cell with the smallest capacity.
- Findings from this study advocates the use of LTM-NDP over CTM-NDP in terms of optimum budget allocation for a network design problem.

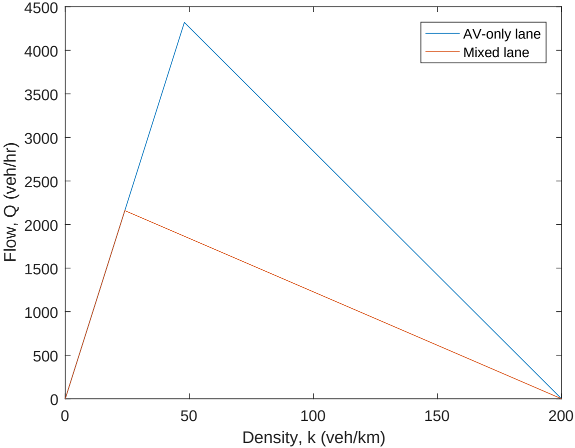
Chakraborty, S., Rey, D., Moylan, E. and Waller, S. T. (2018). Link Transmission Model based Linear Programming Formulation for Network Design, *Transportation Research Record*, 2672(48), 139-147.

Deployment of AV lanes with Endogenous Demand

Deployment of AV lanes with Endogenous Demand

- An integrated mixed-integer programming framework for optimal lane allocation for AVs on a free-way network
- Multi-class LTM is developed with legacy vehicles (LV) and autonomous vehicles (AV).
- Demand is endogenous to the model where mode choice decisions are expressed by a logit model.
- Binary lane allocation variables and logit model for mode choice results in a mixed-integer non-linear program (MINLP)

Fundamental diagram for two lane types



Fundamental diagrams of traffic flow for two lane types

Multi-class Link Transmission Model with dedicated AV lane

Objective function:

$$\min \delta \sum_{i \in A \setminus A_s} \sum_{(o,d) \in K} \sum_{t \in T} \left(z_{i,lv}^{k+}(t) + z_{i,av}^{k+}(t) - z_{i,lv}^{k-}(t) - z_{i,av}^{o,d-}(t) \right)$$

Constraint on restricted AV lanes:

$$\sum_{(o,d) \in K} \sum_{j \in \Gamma^{+}(i)} y_{i,j,lv}^{k}(t) \leq (1 - b_i) M \qquad \forall j \in A \setminus (A_r), \forall t \in T$$

$$\sum_{(o,d) \in K} \sum_{i \in \Gamma^{-}(j)} y_{i,j,lv}^{k}(t) \leq (1 - b_j) M \qquad \forall j \in A \setminus (A_r, A_s), \forall t \in T$$

Mode choice constraints

Average travel time:

$$\tau_{lv} = \frac{\sum_{i\in L}\sum_{k\in A_s}\sum_{t\in T}\left(z_{i,lv}^{k+}(t) - z_{i,lv}^{k-}(t)\right)\delta}{\sum_{i\in L}\sum_{o\in A_r}\sum_{d\in A_s}\sum_{t\in R\setminus t_n} D^{o,d}(t)(1-p^{o,d})a_i^{o,d}} \quad \forall l \in L$$

$$\tau_{av} = \frac{\sum_{i\in L}\sum_{k\in A_s}\sum_{t\in T}\left(z_{i,av}^{k+}(t) - z_{i,av}^{k-}(t)\right)\delta}{\sum_{i\in L}\sum_{o\in A_r}\sum_{d\in A_s}\sum_{t\in R\setminus t_n} D^{o,d}(t)p^{o,d}a_i^{o,d}} \quad \forall l \in L$$

Utility functions:

$$p^{o,d} = \frac{1}{e^{\left(\beta_{\tau_{av}}^{o,d}\tau_{av}^{o,d} - \beta_{\tau_{lv}}^{o,d}\tau_{lv}^{o,d}\right)} + 1} \qquad \forall (o,d) \in K$$

Freeway network design problem

Model ($\mathcal{FNDP}$)

$$\begin{cases} \min \ TSTT \\ \textit{s.t.:} \\ \textit{Endogenous demand constraints} \\ \textit{Network dynamics constraints} \\ \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{p} \end{cases}$$

Benders decomposition method

Model ($\mathcal{SP}(\boldsymbol{b}, \boldsymbol{p})$)

$$\begin{cases} \min \ \mathit{TSTT} \\ \textit{s.t.:} \\ \textit{Network dynamics} \\ \boldsymbol{y}, \boldsymbol{z} \end{cases}$$

Model ($\mathcal{MP}$)

$$\begin{cases} \mathcal{MP}\text{: } \min \ Z \\ \textit{s.t.:} \\ Z \geq \textit{Optimality cuts} \\ 0 \geq \textit{Feasibility cuts} \\ \boldsymbol{b} \in \{0, 1\} \end{cases}$$

Fixed Point Algorithm

$\boldsymbol{p}^0 \leftarrow \frac{1}{e^{\tau_{ff}^{o,d}\left(\beta_{\tau_{av}}^{o,d} - \beta_{\tau_{lv}}^{o,d}\right)}+1}$ (Solve logit model with free-flow travel times)

$\boldsymbol{b}^0 \leftarrow \mathbf{0}$ (No AV lanes)

$n \leftarrow 0$

repeat

$n \leftarrow n+1$

$\boldsymbol{y}^n, \boldsymbol{z}^n \leftarrow$ Solve $\mathcal{SP}(\boldsymbol{b}^m, \boldsymbol{p}^n)$

for $(o,d) \in K$ **do**

$$\tau_{lv}^{o,d,n} \leftarrow \frac{\sum_{i \in A \setminus A_s} \sum_{t \in T} \left(z_{i,lv}^{k+}(t) - z_{i,lv}^{k-}(t) \right) \delta}{D^{o,d}(t)(1-p^{o,d})}$$

$$\tau_{av}^{o,d,n} \leftarrow \frac{\sum_{i \in A \setminus A_s} \sum_{t \in T} \left(z_{i,av}^{k+}(t) - z_{i,av}^{o,d-}(t) \right) \delta}{D^{o,d}(t) p^{o,d}}$$

$$p^{o,d} \leftarrow \frac{1}{e^{\left(\beta_{\tau_{av}}^{o,d} \tau_{av}^{o,d} - \beta_{\tau_{lv}}^{o,d} \tau_{lv}^{o,d}\right)}+1}$$

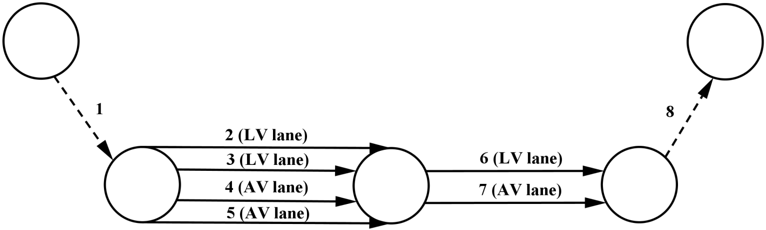
$$p_{n+1}^{o,d} \leftarrow \frac{n}{n+1} p_n^{o,d} + \frac{1}{n+1} p^{o,d}$$

until $\sum_{(o,d) \in K} |p_{n+1}^{o,d} - p_n^{o,d}| \leq \epsilon_{MSA}$;

Benders Decomposition Method

$m \leftarrow 0$
$UB \leftarrow \infty$
repeat
  $m \leftarrow m + 1$
  $\boldsymbol{y}^n, \boldsymbol{z}^n \leftarrow$ Solve $\mathcal{SP}(\boldsymbol{b}^m, \boldsymbol{p}^n)$ and Fixed point algorithm
  if $\mathcal{SP}(\boldsymbol{b}^m, \boldsymbol{p}^n)$ *is infeasible* **then**
    Generate feasibility cut
  else
    if $TSTT^n < UB$ **then**
      $UB \leftarrow TSTT^n$
      $\boldsymbol{b}^\star \leftarrow \boldsymbol{b}^m$
      $\boldsymbol{\tau}^\star \leftarrow \boldsymbol{\tau}^n$
      $\boldsymbol{p}^\star \leftarrow \boldsymbol{p}^n$
    if $GAP > \epsilon$ **then**
      Generate optimality cut
  $\boldsymbol{b}^m, Z^m \leftarrow$ Solve $\mathcal{MP}$
until $GAP \leq \epsilon$;
return $UB, \boldsymbol{b}^\star, \boldsymbol{p}^\star, \boldsymbol{\tau}^\star$

Existence and non-uniqueness of fixed points

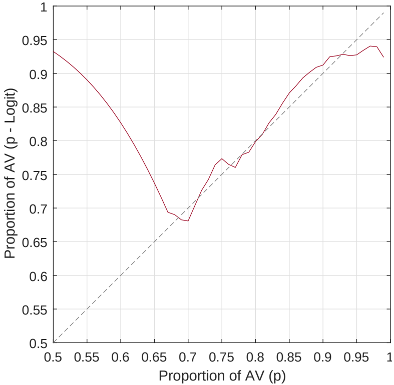


Single OD case study network

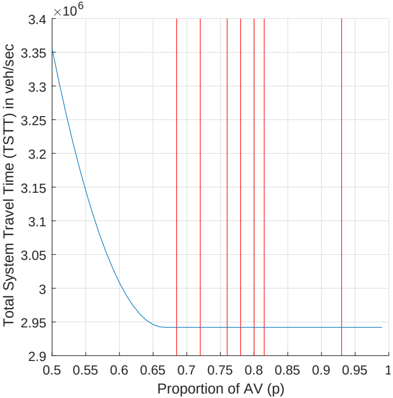
Network characteristics

Parameters	Source connector	Lane 2	Lane 3	Lane 4	Lane 5	Lane 6	Lane 7	Sink connector
Length (km)	0.0001	0.8	0.8	0.8	0.8	0.8	0.8	0.0001
Free-flow speed (kmph)	90	90	90	90	90	90	90	90
Backward wave speed (kmph)	12.28	12.28	12.28	28.42	28.42	12.28	28.42	12.28
Capacity (veh/hr)	360000	2160	2160	4320	4320	2160	4320	360000
Jam density (veh/km)	100000	200	200	200	200	200	200	100000

Existence and non-uniqueness of fixed points

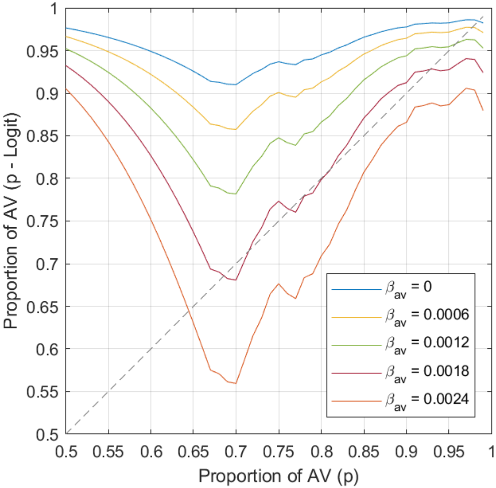


Multiple fixed points
(with fixed lane allocation)



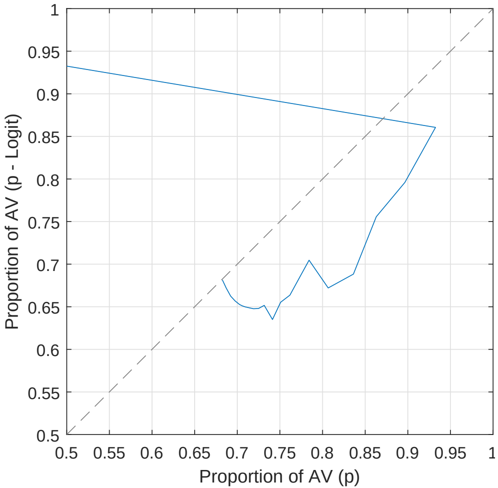
Change in $TSTT$ with p

Existence and non-uniqueness of fixed points



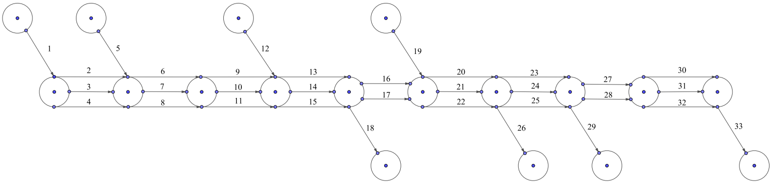
Sensitivity analysis with different $\beta_{\tau_{av}}^{o,d}$ values

Fixed point algorithm



Convergence of FP algorithm
(with fixed lane allocation)

Multi-OD Freeway Network



Performance of the proposed algorithm for different scenarios

Parameters	Base	-25% Demand	+25% Demand	-25% q_{av}	+25% q_{av}	-25% β_{av}	+25% β_{av}
Nb of OD pairs	6	6	6	6	6	6	6
Nb of converted AV lanes	7(9)	8(9)	6(9)	6(9)	8(9)	7(9)	7(9)
CPU time (mins)	44.93	70.38	44.12	21.32	96.06	32.20	40.82
Nb of Benders iterations	29	68	22	10	76	26	28
Nb of FP iterations	172	151	166	86	363	111	127
TSTT (veh-sec)	2125200	1450710	2984970	2277540	2026440	2125200	2125560
Reduction in TSTT (%)	16.30	9.50	19.45	10.30	20.19	16.30	16.29

Effect of AV-only lanes on average travel times of LVs

Links	Base		-25% Demand		+25% Demand		-25% q_{av}	+25% q_{av}	-25% β_{av}	+25% β_{av}
	Initial	Optimal	Initial	Optimal	Initial	Optimal	Optimal	Optimal	Optimal	Optimal
1	262.66	182.70	132.69	93.38	396.92	149.78	176.40	86.51	80.07	155.07
2	203.99	144.05	172.12	129.16	252.60	232.08	187.53	139.72	173.76	141.24
3	313.67	171.32	185.01	106.72	411.28	221.52	193.41	204.24	157.82	190.71
4	141.69	132.08	129.34	136.94	145.85	147.59	135.56	125.96	148.51	131.02
5	136.66	135.06	126.87	137.07	142.22	142.90	141.28	135.16	136.24	137.55
6	206.93	157.22	116.03	80.75	394.33	239.85	159.26	110.57	182.75	121.79
7	139.61	125.56	124.50	125.32	140.34	122.76	125.94	125.22	125.24	122.60
8	135.20	128.65	123.46	120.81	128.78	135.79	129.59	120.24	122.27	123.37
10	297.89	146.23	143.11	88.99	386.22	291.27	208.81	113.66	137.85	139.23
11	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00
12	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00
14	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00
16	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00

Effect of AV-only lanes on average travel times of AVs

Links	Base		-25% Demand		+25% Demand		-25% q_{av}	+25% q_{av}	-25% β_{av}	+25% β_{av}
	Initial	Optimal	Initial	Optimal	Initial	Optimal	Optimal	Optimal	Optimal	Optimal
1	163.00	141.02	111.39	76.14	317.90	187.83	182.10	129.84	144.71	120.59
2	167.32	157.95	155.05	139.76	251.77	177.40	146.48	136.26	148.99	155.26
3	220.45	179.83	148.31	98.18	335.50	257.70	222.09	141.06	178.18	167.25
4	144.56	132.95	152.82	129.89	173.21	138.53	138.36	128.83	126.87	137.94
5	135.59	123.65	138.41	125.76	152.82	129.31	126.30	127.49	128.82	129.22
6	248.83	95.30	148.33	88.23	209.56	145.76	156.64	85.72	131.23	119.30
7	139.90	123.26	123.81	121.34	153.85	123.47	123.61	124.04	123.44	123.86
8	133.21	130.79	121.31	121.75	146.63	134.21	138.23	120.74	131.25	128.66
10	195.01	112.43	135.63	79.05	376.81	202.08	167.18	87.80	113.73	122.24
11	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00
12	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00
14	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00
16	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00

Conclusion

Conclusion

- Interaction of AVs and LVs are inevitable in near future
- It is crucial to find out the critical locations of a network where resource allocation will reap maximum system level benefits
- Fixed point algorithm reduces the proposed formulation to a tractable optimization program
- We prove that the proposed solution method converges to a local optima of the nonconvex problem
- We identify under which conditions this local optima is a global solution.
- The proposed model is flexible enough to incorporate sophisticated utility functions for mode choice decisions made by commuters with different mode preferences.

Thank you