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TransportLab

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A Multi-stage Spatial Queueing Model with Logistic Arrivals and Departures Consistent with the Microscopic Fundamental Diagram and Hysteresis

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Research Question

- **Traffic state estimation** involves the prediction of **congestion formation** and **dissipation processes** (Wang and Papageorgiou, 2005), and **accurate estimation** of **traffic state** has become indispensable for effective traffic management and decision-making (Seo et al., 2017).

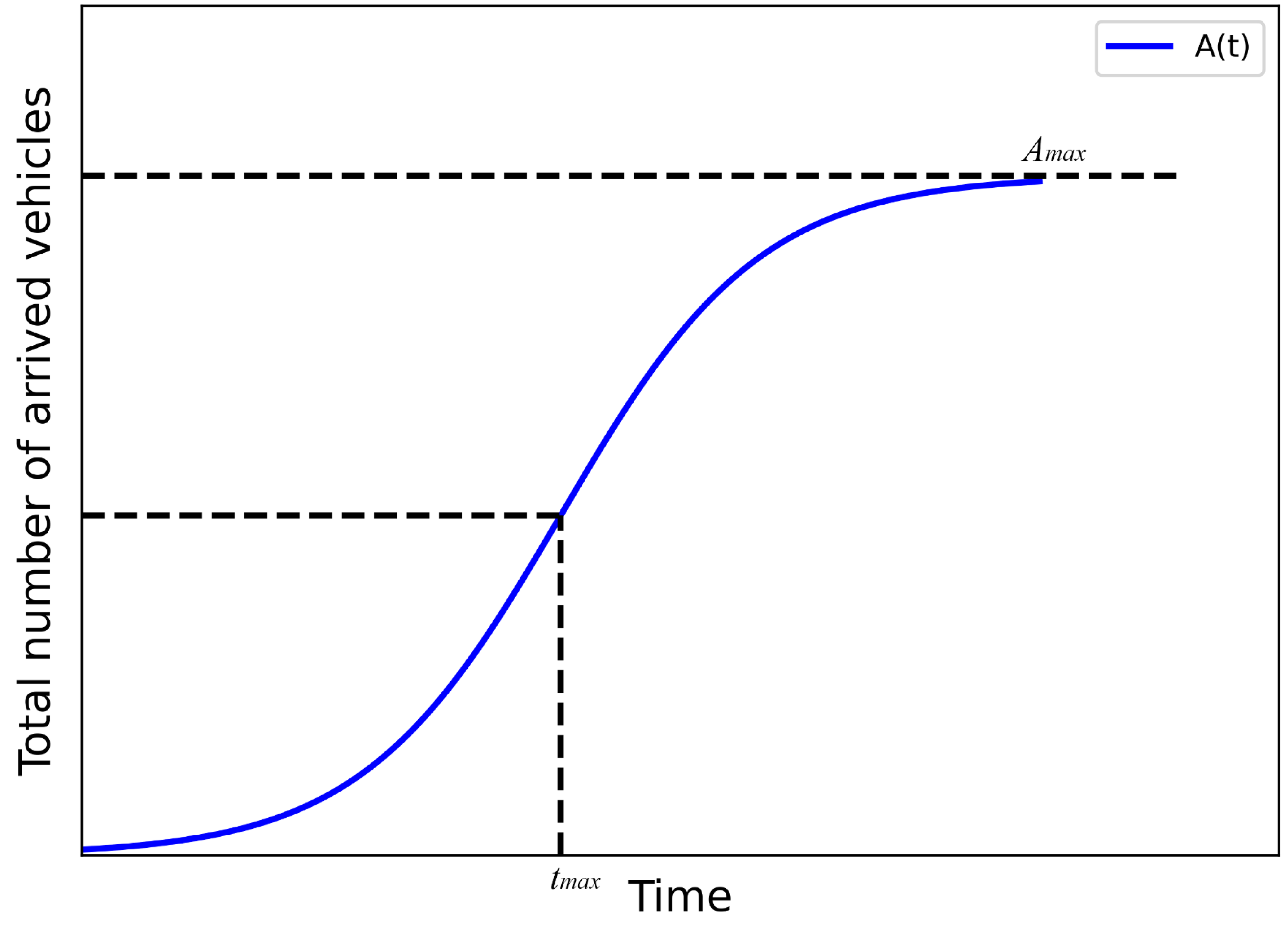
- **Q1 :** How to **accurately** describe the **arrival** and **departure** of traffic **flow** during the **morning peak period** in a **continuous** and **differentiable** form?

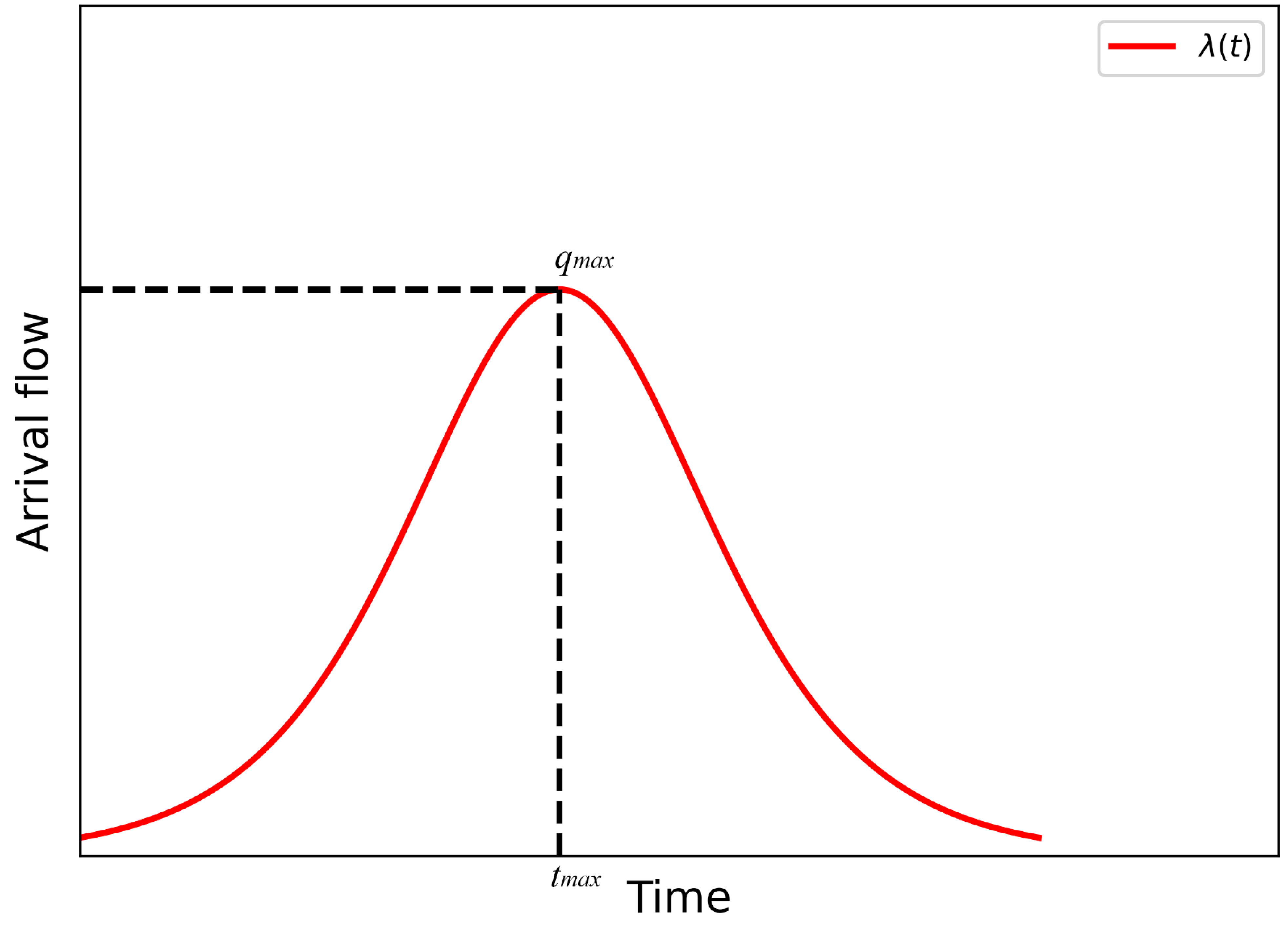
- **H1 :** The **logistic function** can accurately describe the **characteristics** of **morning peak flow** without losing **good mathematical properties**.

- **Q2 :** How to describe the **queueing process** considering the **different traffic states in space**?

- **H2 :** Considering the three different states of freeflow, transition, and queued in the link, it can ensure the **conservation of flow mass** and achieve **spatial continuity** at the same time.

Logistic model for arrival flow function



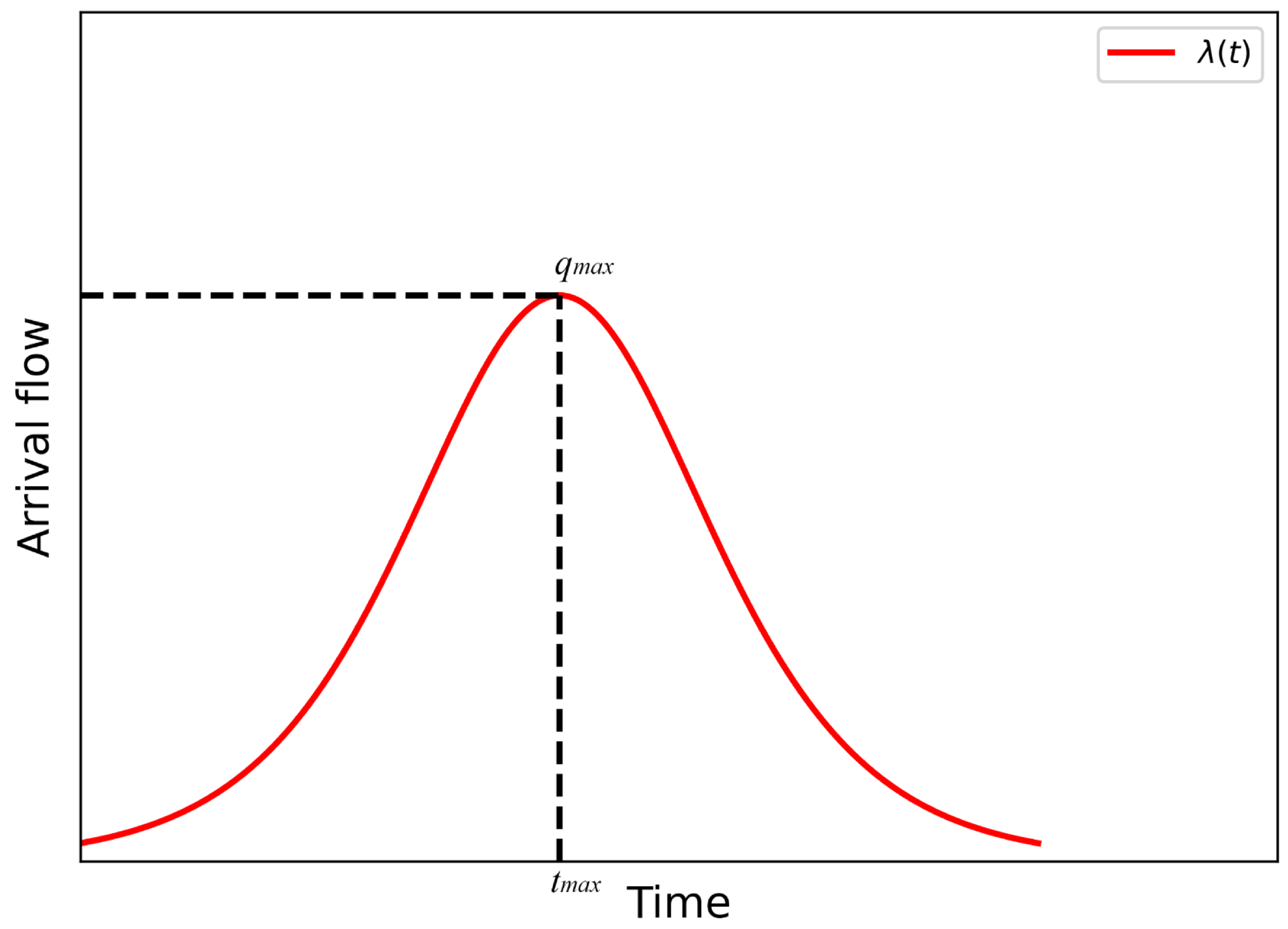


Cumulative number of arrived vehicle $A(t) = \frac{A_{max}}{1 + e^{-\theta_a (t - t_{max})}}$

Arrival flow function $\lambda(t) = \frac{dA(t)}{dt} = 4 \cdot q_{max} \cdot \frac{e^{\theta_a (t - t_{max})}}{(1 + e^{\theta_a (t - t_{max})})^2}$

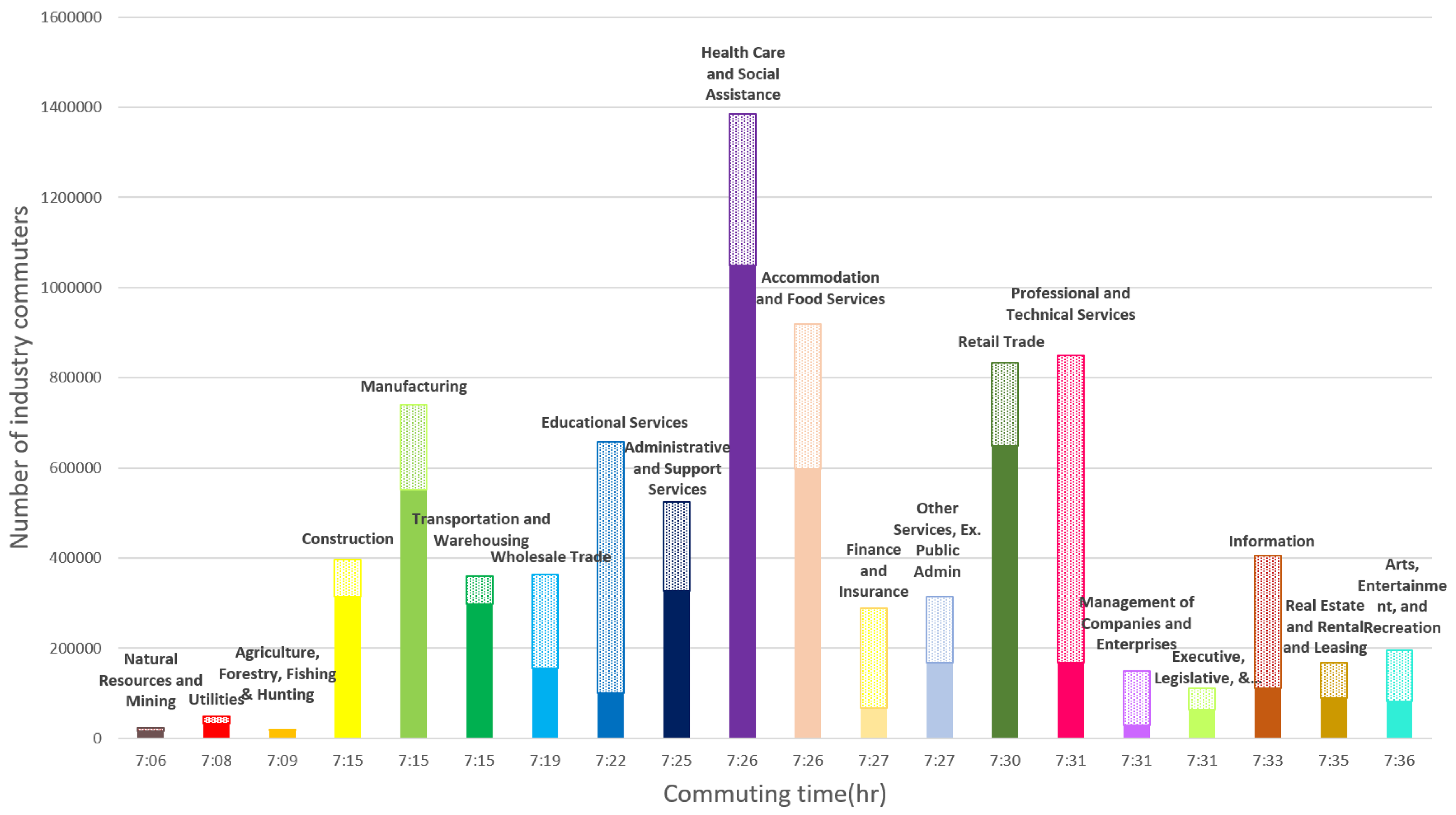
Where A_{max} is the **maximum** value of the **arrived vehicles**,
θ_a is the **growth rate** of arriving vehicles,
t_{max} is the **time** when the **arriving vehicles** increase fastest, that is, the time corresponding to the **maximum value** of the **first-order derivative**,
q_{max} is the **maximum** value of the **arrival flow**, which is equal to $\frac{1}{4} \cdot A_{max} \cdot \theta_a$.

Logistic model for arrival flow function

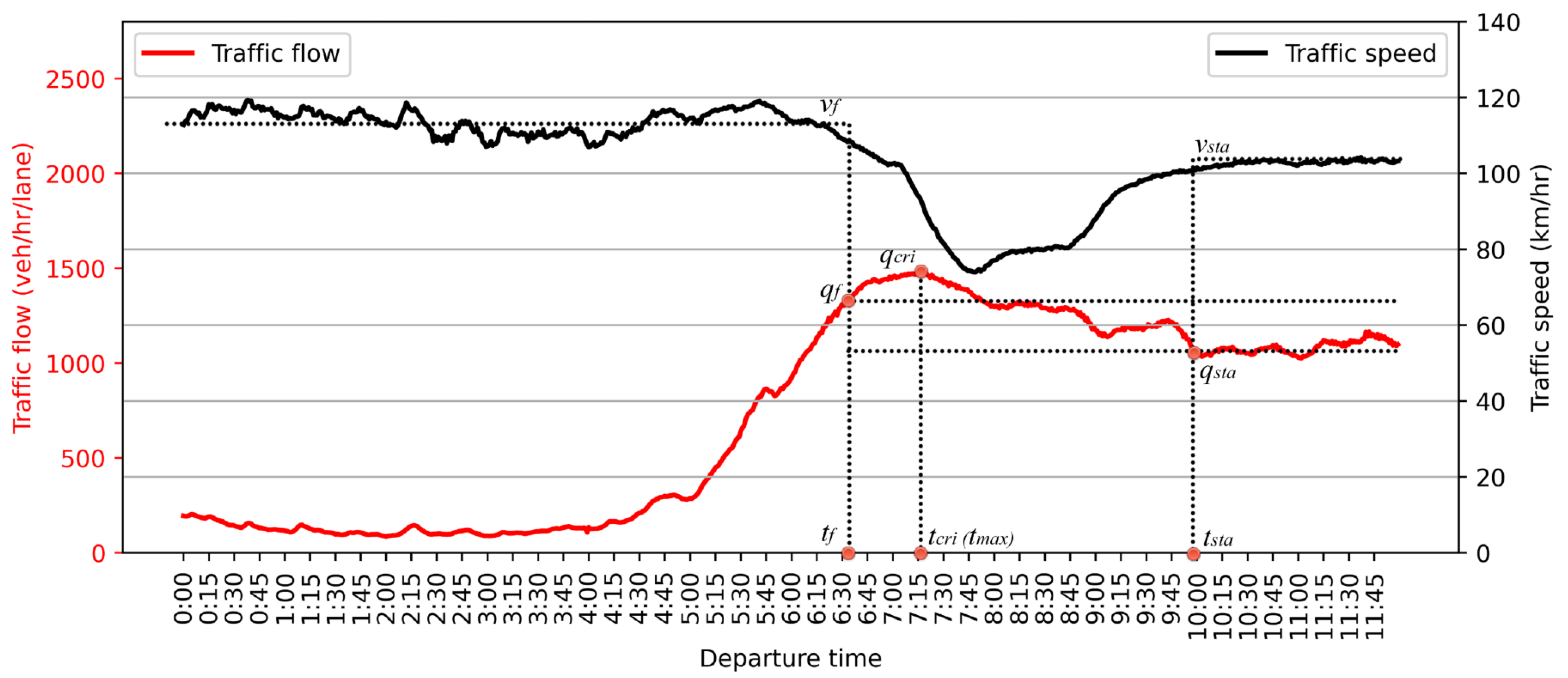


Arrival flow function $\lambda(t)$

- Peak commuting only?
- Does not include non-work trips

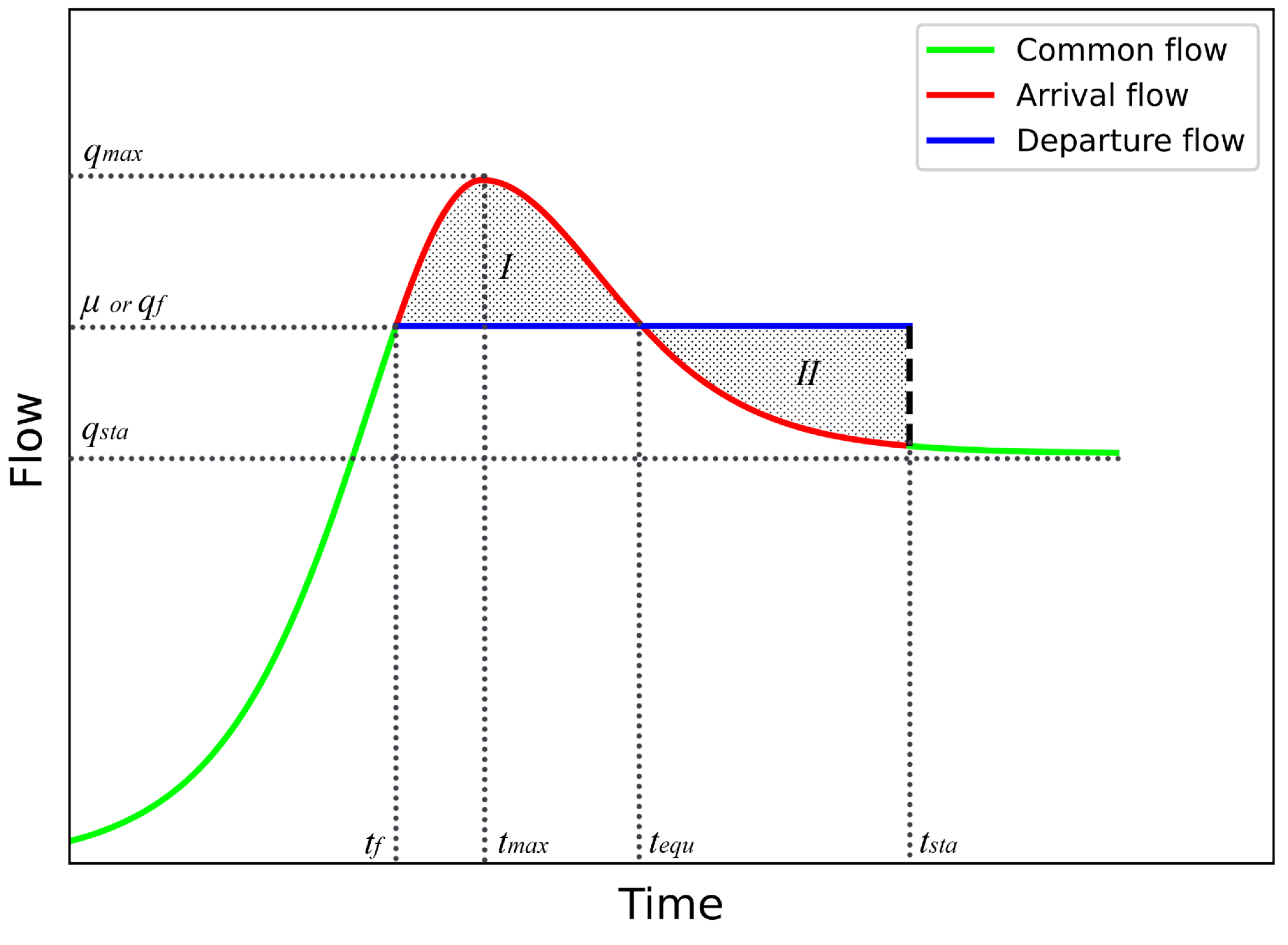


The **distribution** of **morning peak commuters** across the California in **time** and **industry**



The **diurnal curves** of the **traffic flow** and **speed** of the freeway stretch I-694 (W-E) from **12:00 midnight to 12:00 noon**

Logistic model for arrival flow function



Arrival flow function $\lambda_c(t)$

$$\lambda_c(t) = \begin{cases} 4 \cdot q_{max} \cdot \frac{e^{\theta_a(t-t_{max})}}{(1+e^{\theta_a(t-t_{max})})^2} & , t \leq t_{max} \\ 4 \cdot (q_{max} - q_{sta}) \cdot \frac{e^{\theta_a(t-t_{max})}}{(1+e^{\theta_a(t-t_{max})})^2} + q_{sta} & , t > t_{max} \end{cases}$$

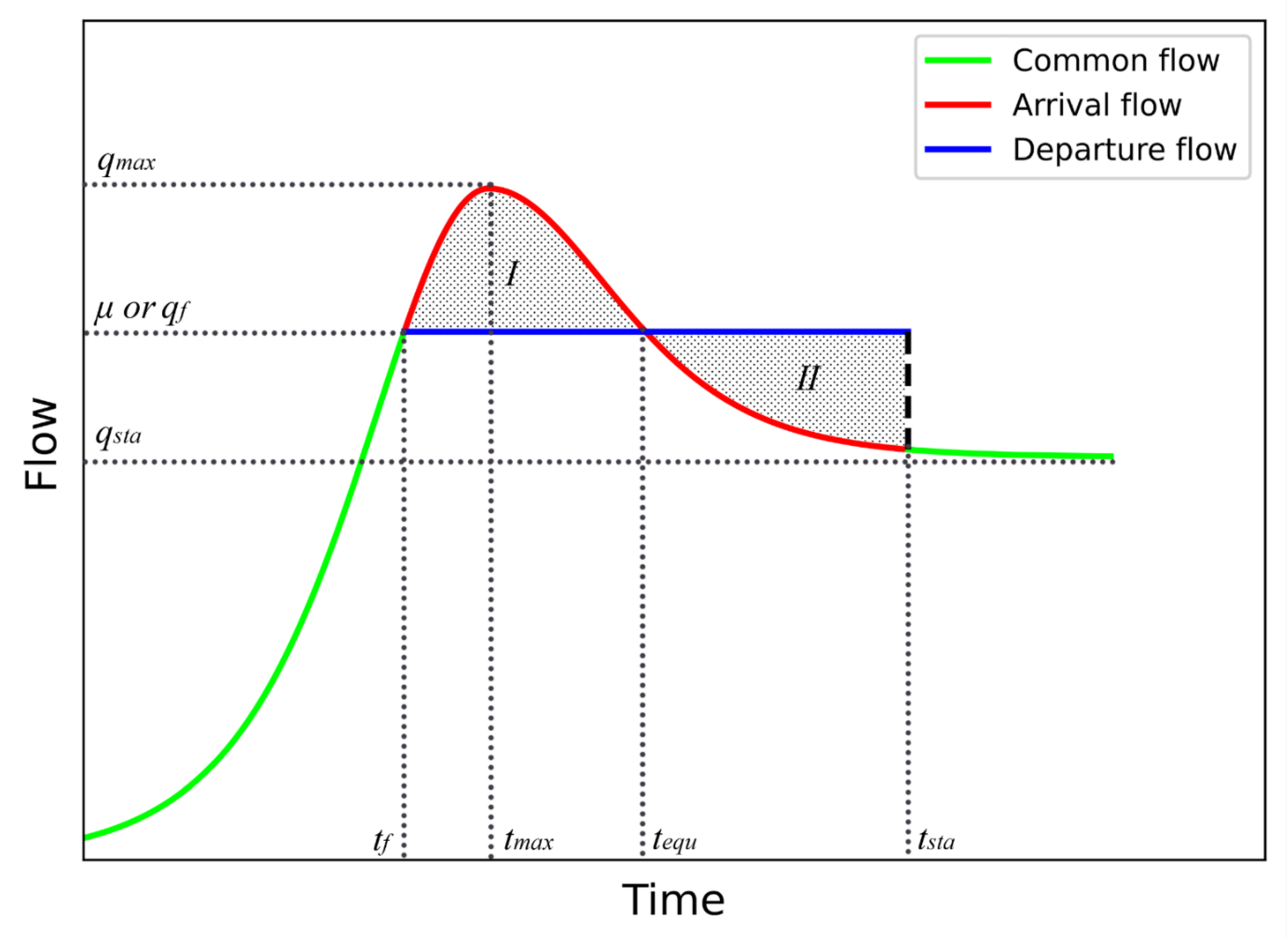
Where q_{sta} is the arrival flow after the queue dissipation and stabilized, which is smaller than q_f.

$$\lim_{t \to t_{max}^-} \lambda_c(t) = \lim_{t \to t_{max}^+} \lambda_c(t) = q_{max}$$

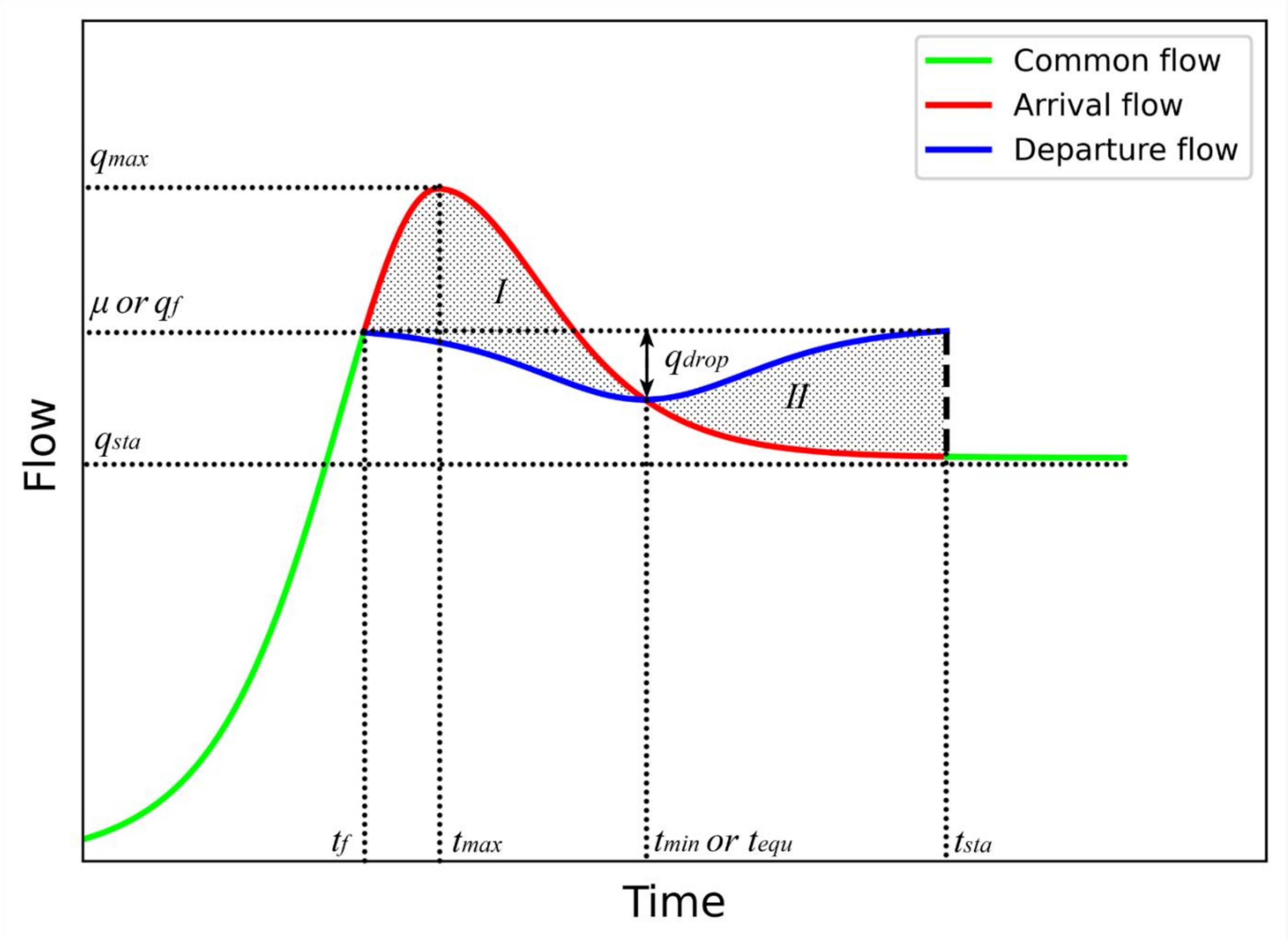
$$\lim_{t \to t_{max}^-} \frac{d\lambda_c(t)}{dt} = \lim_{t \to t_{max}^+} \frac{d\lambda_c(t)}{dt} = 0$$

Where A_{max} is the **maximum** value of the **arrived vehicles**,
θ_a is the **growth rate** of arriving vehicles,
t_{max} is the **time** when the **arriving vehicles** increase fastest,
q_{max} is the **maximum** value of the **arrival flow**, which is equal to $\frac{1}{4} \cdot A_{max} \cdot \theta_a$.

Logistic model for departure flow function



Departure flow function with **constant capacity μ**



Departure flow function with **varying capacity $\mu_v(t)$**

$$q_{dep}(t) = \begin{cases} \mu & , N(t) > 0 \\ \lambda_c(t) & , N(t) = 0 \end{cases}$$

Where $N(t) = \int_{t_f}^{t} \lambda_c(t) - \mu dt$

$$\mu_v(t) = \mu - 4 \cdot q_{drop} \cdot \frac{e^{\theta_d(t - t_{min})}}{(1 + e^{\theta_d(t - t_{min})})^2} + \epsilon$$

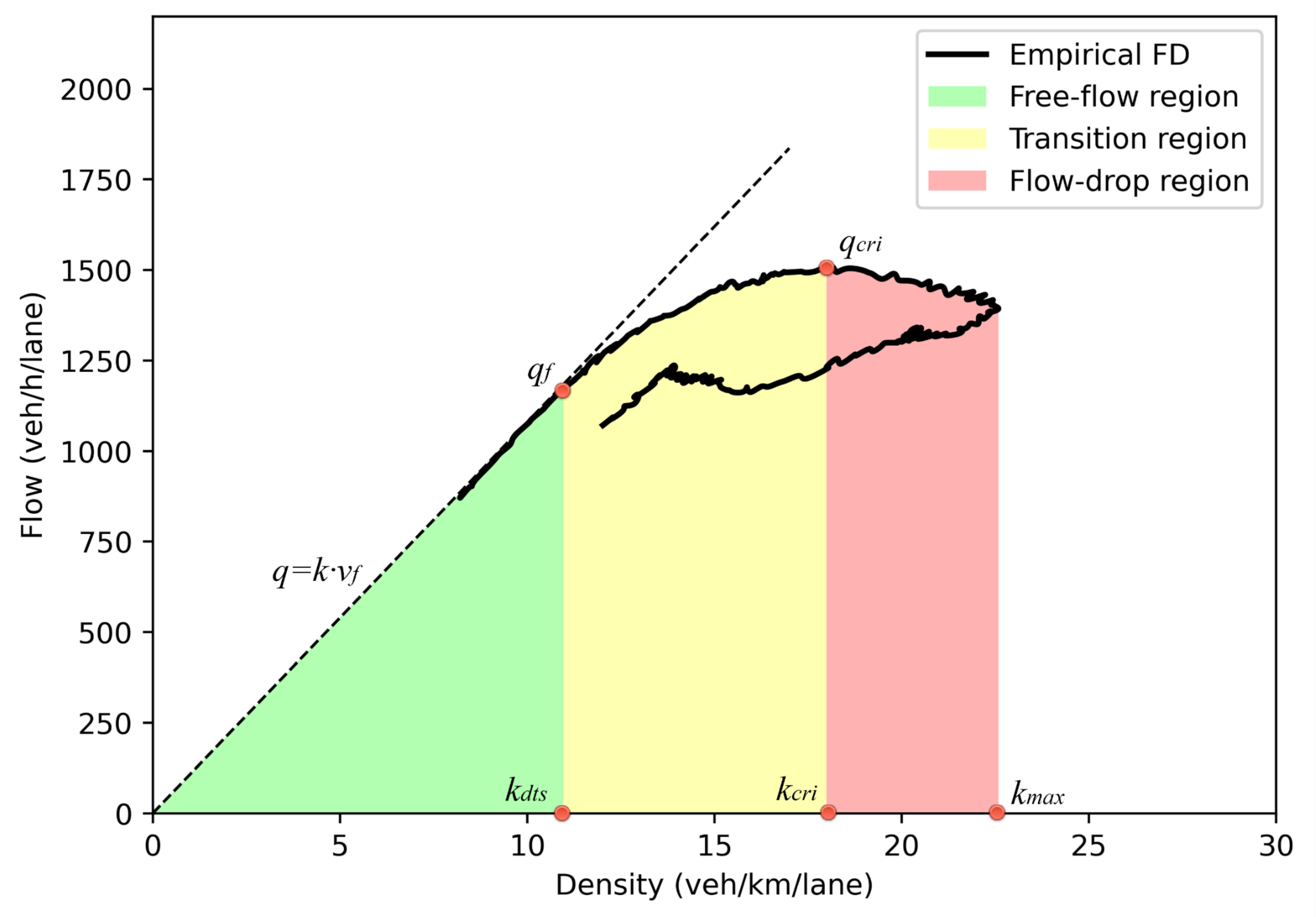
Where q_{drop} is the **upper limit** of the **capacity loss**,

t_{min} is the **occurrence time** of the minimum capacity,

θ_d is the **decay rate** of departing vehicles.

ϵ is a minor compensation term, which is used to ensure the **continuity** of the function.

Definition of traffic density of the link



The **empirical FD** of the freeway stretch **I-694 (W-E)** in the **MSP** freeway network

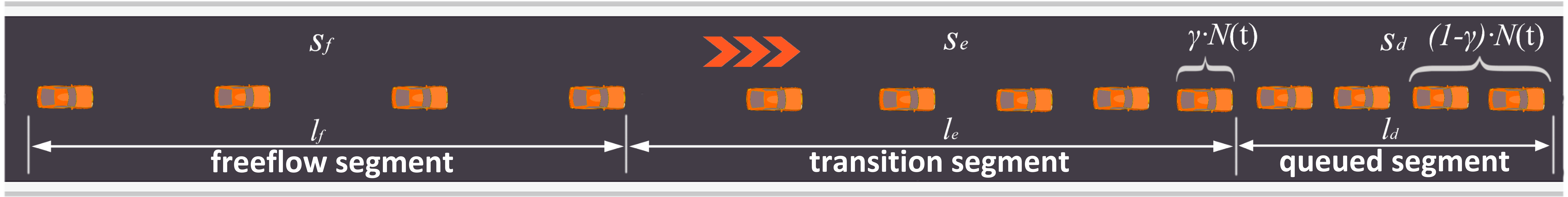
$$k(t) = \frac{\frac{\lambda_c(t)}{v_f}\cdot l + N(t)}{l} = \frac{\lambda_c(t)}{v_f} + \frac{N(t)}{l}$$

Where l is the **length** of the **link**, and v_f is the **freeflow speed**.

Queueing considering expansion and spillback

- Most previous studies (Lu et al., 2023, Newell, 1988, Zhou et al., 2022) **ignore** the **physical length** of vehicles and assume that **accumulated vehicles** are stacked on the **head of the link**.

- In this paper, we assume that the **length** of the **queued segment** s_d and the number of **accumulated vehicles** $N(t)$ satisfy the relationship:



$$l_d(t) = \left(\frac{N(t)}{k_{dts}}\right)^{\gamma} = \xi \cdot N(t)^{\gamma}$$

Where $\gamma = \frac{k_{dts}}{k_{max}}$, $\xi = k_{dts}^{-\gamma}$

k_{dts} is the **maximum density** that the link **maintains freeflow state**,

k_{max} **is maximum average vehicle density** of the link.

Queueing considering expansion and spillback

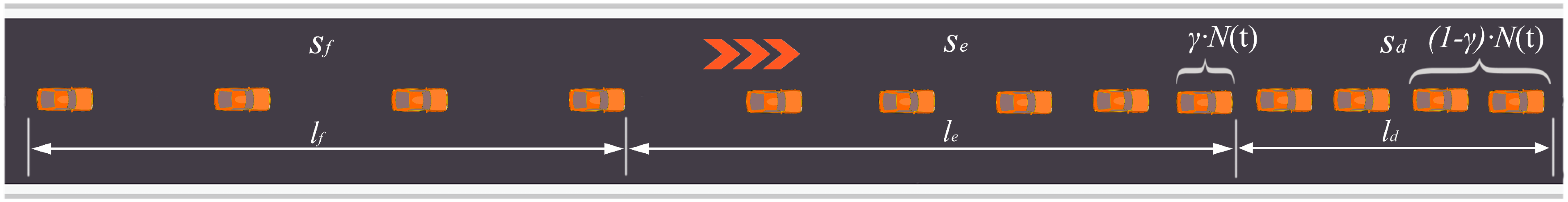
- The **density** of the **freeflow segment** s_f satisfy the relationship:

$$k_f(t) = \frac{\lambda_c(t)}{v_f}$$

This is just ***Q=KV***

- We assume that some vehicles **spillback** from **queued segment** s_d to **transition segment** s_e:

$$N_s(t) = \gamma \cdot N(t)$$

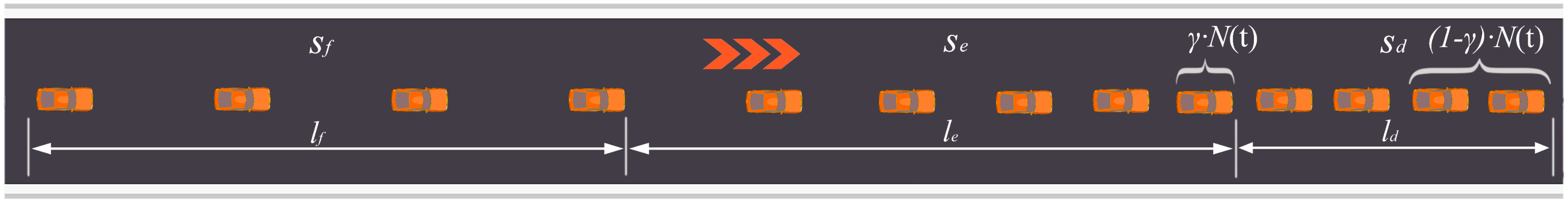


Queueing considering expansion and spillback

- The **density** of the **queued segment** s_d satisfy the relationship:

$$k_d(t) = \frac{\frac{\lambda_c(t)}{v_f} \cdot l_d(t) + (1-\gamma) \cdot N(t)}{l_d(t)} = \frac{\lambda_c(t)}{v_f} + \frac{(1-\gamma) \cdot N(t)^{1-\gamma}}{\xi}$$

- The **density** $k_e(x,t)$ of the **transition segment** s_e increases linearly from $k_f(t)$ to $k_d(t)$, which is used to connect two different traffic states upstream and downstream to ensure spatial continuity.



Travel time of the queued, transition, and freeflow segments

- Based on the condition of **density homogeneity** and **flow linear variation** in the **queued segment** s_d, its **travel time** $w_d(t)$ can be indicated as:

$$w_d(t) = \int_0^{l_d(t)} \frac{1}{v(x)} dx$$
$$= \int_0^{l_d(t)} \frac{k_d(t)}{\lambda_c(t) + \Delta q_d \cdot x} dx$$
$$= \frac{k_d(t) \cdot l_d(t) \cdot ln\left(\frac{\mu}{\lambda_c(t)}\right)}{\mu - \lambda_c(t)}$$

- It can be proved when $\mu = \lambda_c(t)$, $w_d(t) = \frac{k_d(t) \cdot l_d(t)}{\lambda_c(t)}$, which means that the **queued segment** is in a state where both **flow** and **density** are **homogeneous**.

Travel time of the queued, transition, and freeflow segments

- For the **length** $l_e(t)$ of the **transition segment** s_e, it can be determined according to the conservation between the number of spillback vehicles and the increase in the overall density of s_e.

$$N_s(t) = \int_0^{l_e(t)} k_e(x,t) - k_f(t)\mathrm{d}x$$

$$l_e(t) = \frac{2 \cdot \gamma \cdot \xi \cdot N(t)^\gamma}{1-\gamma}$$

- Then, the **travel time** $w_e(t)$ of **transition segment** s_e can be indicated as:

$$\begin{aligned} w_\mathrm{e}(t) &= \int_0^{l_\mathrm{e}(t)} \frac{1}{v(x)}\mathrm{d}x \\ &= \int_0^{l_\mathrm{e}(t)} \frac{k_f(t) + \Delta k_e \cdot x}{\lambda_c(t)}\mathrm{d}x \\ &= \frac{\left(k_f(t) + k_d(t)\right) \cdot l_e(t)}{2 \cdot \lambda_c(t)} \end{aligned}$$

Travel time of the queued, transition, and freeflow segments

- For **freeflow segment** s_f, the length $l_f(t)$ and **travel time** $w_f(t)$ are:

$$l_f(t) = l - l_d(t) - l_e(t)$$

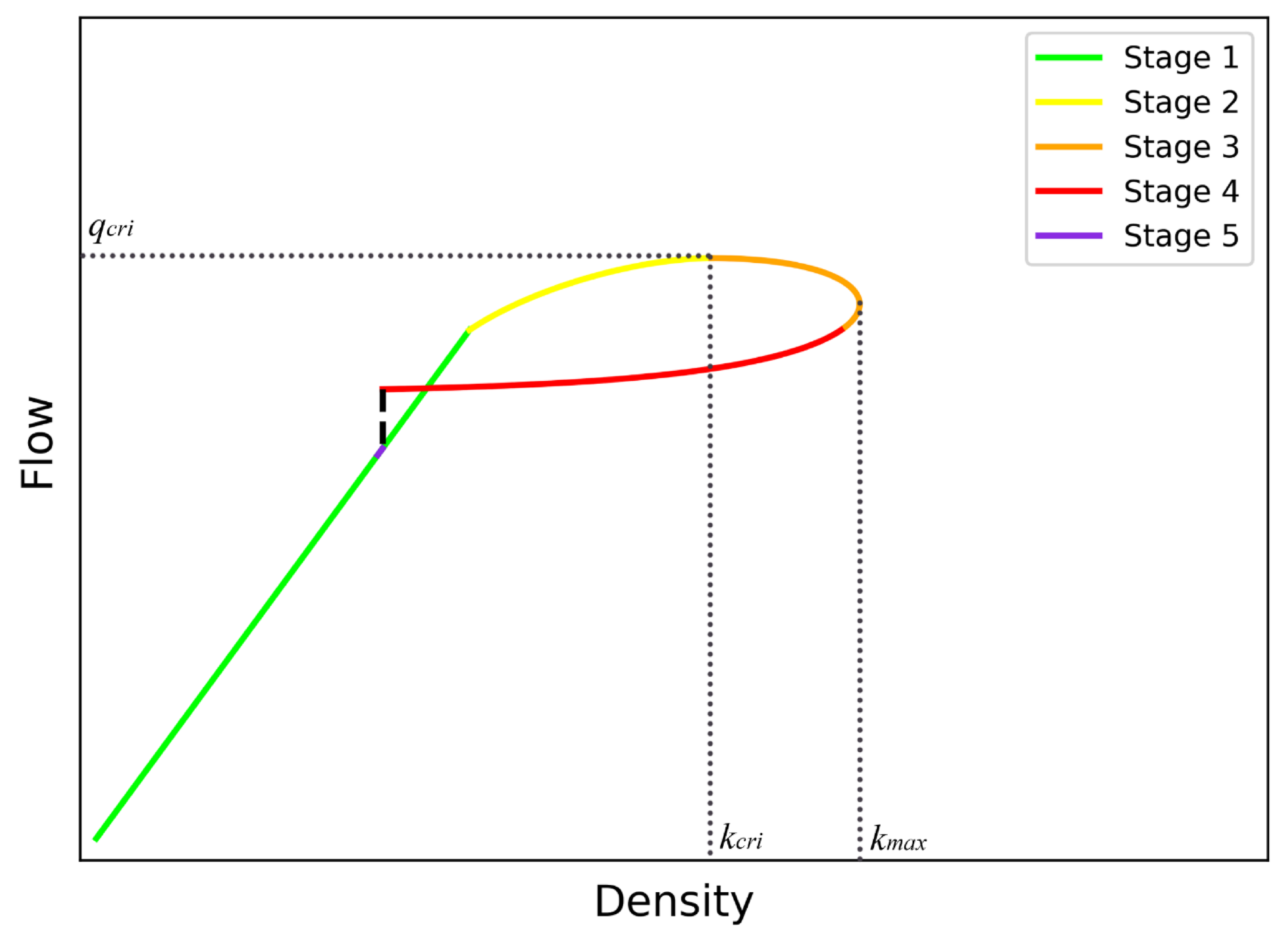
$$w_f(t) = \frac{l_f(t)}{v_f}$$

- Therefore, the **total travel time** of the **entire link** is:

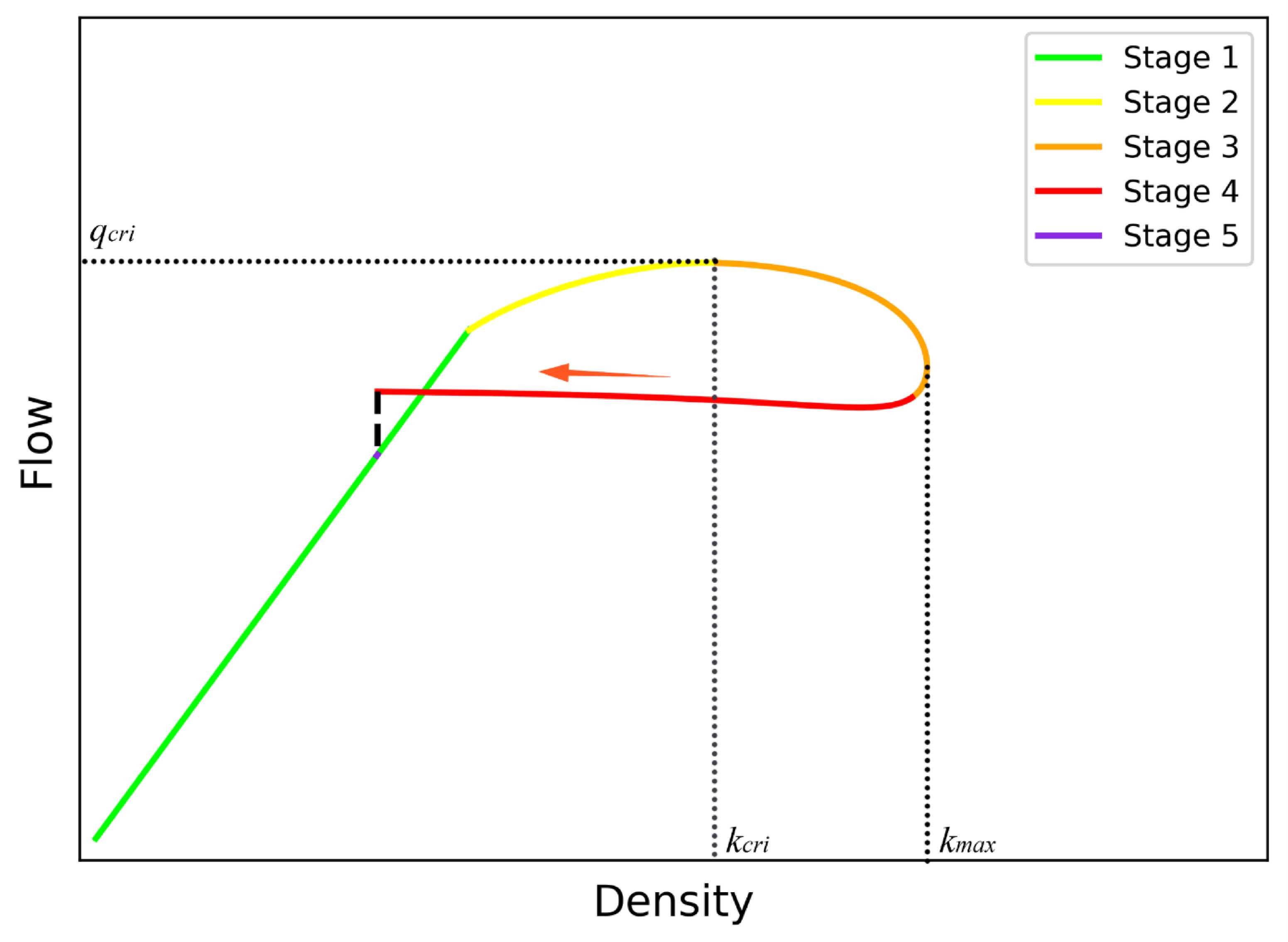
$$W(t) = w_d(t) + w_e(t) + w_f(t)$$
$$= \frac{k_d(t) \cdot l_d(t) \cdot ln\left(\frac{\mu}{\lambda_c(t)}\right)}{\mu - \lambda_c(t)} + \frac{\left(k_f(t) + k_d(t)\right) \cdot l_e(t)}{2 \cdot \lambda_c(t)} + \frac{l_f(t)}{v_f}$$

Reproduction of key features in FDs based on the model

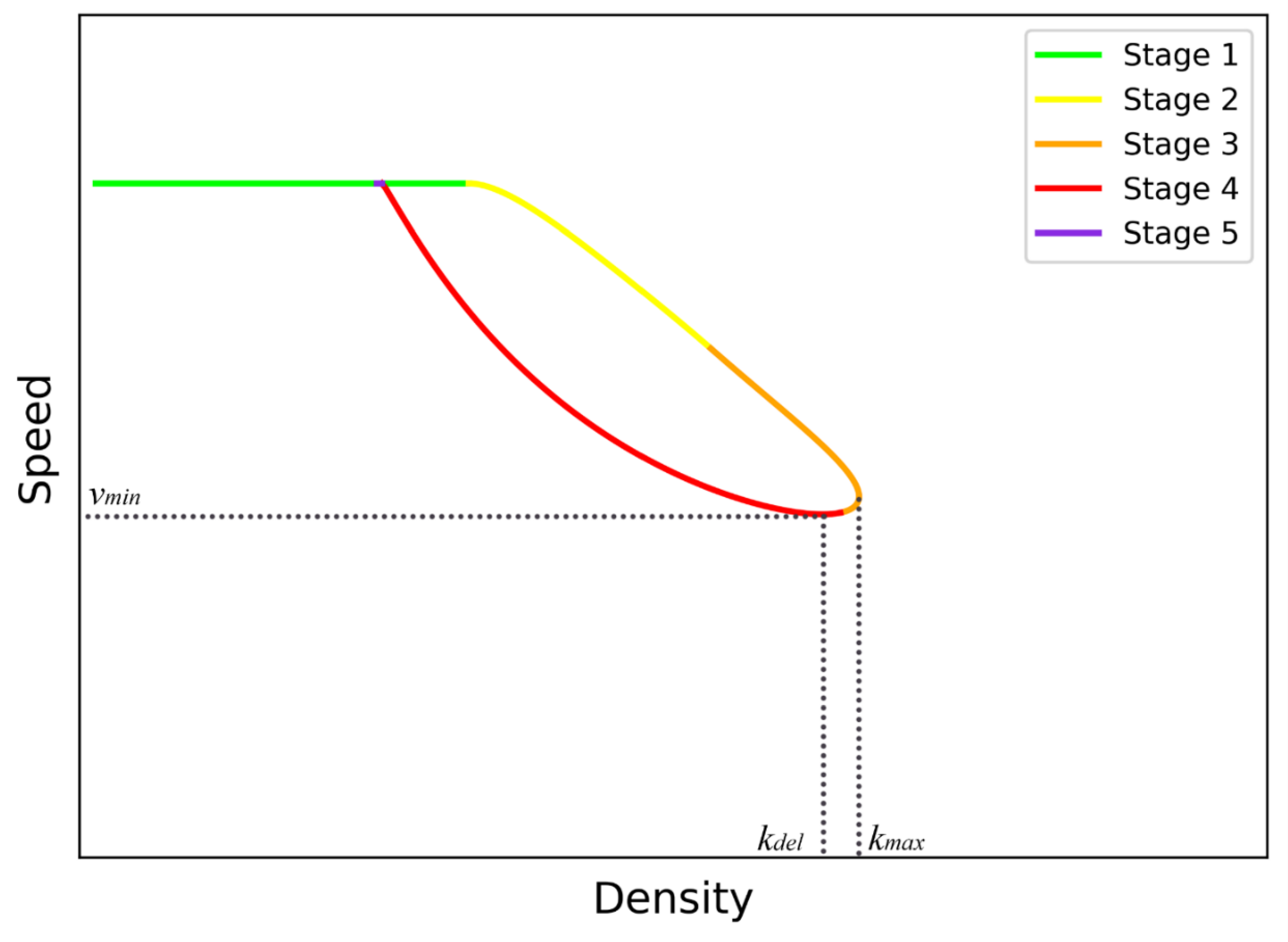
- The **FDs** obtained based on the proposed model:



Fundamental diagram (**constant capacity**)

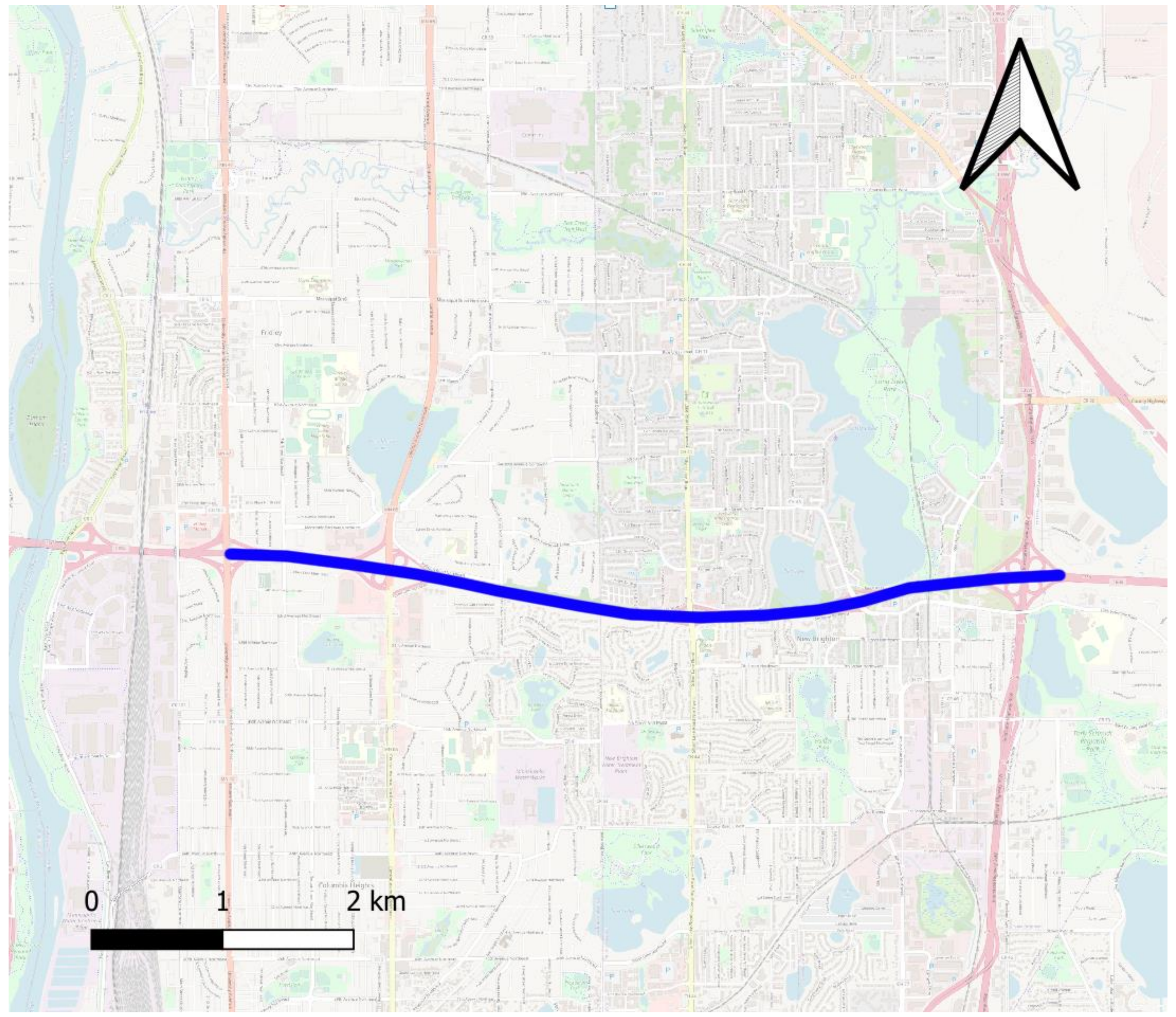


Fundamental diagram (**varying capacity**)

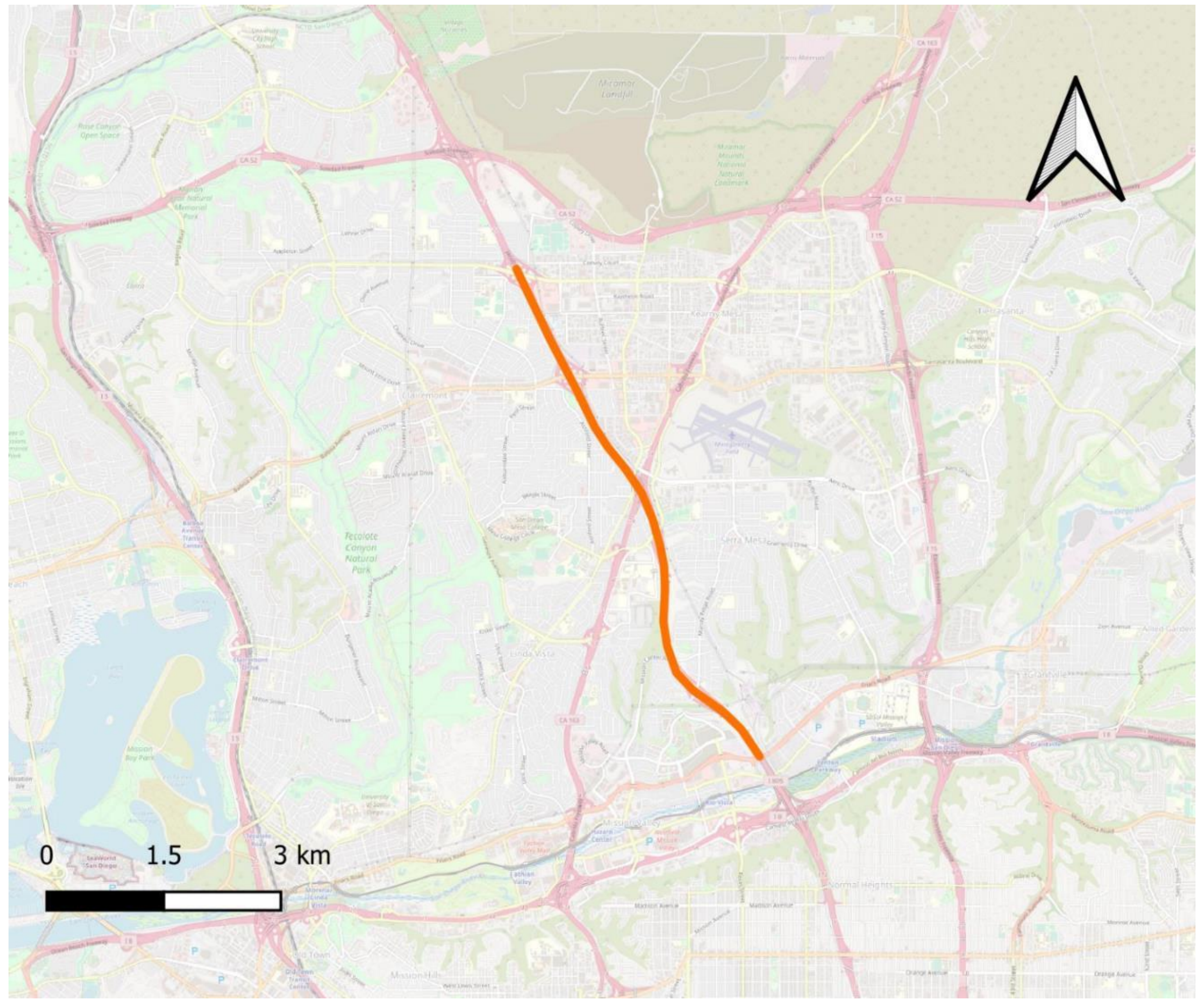


Fundamental diagram (**speed and density under constant capacity**)

Model parameters calibration



MSP freeway network



San Diego freeway network

	I-694 (W-E)	I-805 (S-N)
Source	MSP freeway network	San Diego freeway network
Length	6.35 km	6.54 km
No. of loop detectors	9	10

Model parameters calibration

- For parameter θ_a , that is, the growth rate of the arrival flow, and θ_d and q_{drop} , which are from departure flow model with varying capacity, they can be determined by minimizing the **residual sum of squares** (RSS):

$$\theta_a^* = \arg\min_{\theta_a} \sum_{t=1}^{T} (q(t) - \hat{q}(t))^2$$

$$\theta_a^*, \theta_d^*, q_{drop}^* = \arg\min_{\theta_a, \theta_d, q_{drop}} \sum_{t=1}^{T} (q(t) - \hat{q}(t))^2$$

Where $q(t)$ and $\hat{q}(t)$ are the **estimated** and **real traffic flow** at time interval t obtained from the model and real data, respectively, and $q(t)$ is equal to $\frac{\lambda_c(t)+q_{dep}(t)}{2}$, T is the number of time intervals.

Model parameters calibration

- In addition, considering that the selected freeway stretches include **on-ramps** and **off-ramps**, we add the parameter α to represent the general impact of ramp flow:

$$\lambda_r(t) = \alpha \cdot \lambda_c(t)$$

Where

$$\alpha = \frac{q_{ave}+\sum_{i=1}^{I} q_{on,i}-\sum_{j=1}^{J} q_{off,j}}{q_{ave}}$$

q_{ave} is the average traffic flow of the mainline,
$q_{on,i}$ is the average flow of the $\boldsymbol{i}$ th on-ramp, $\boldsymbol{I}$ is the number of on-ramps, $q_{off,j}$ is the average flow of the $\boldsymbol{j}$ th off-ramp, $\boldsymbol{J}$ is the number of off-ramps.

Model parameters calibration

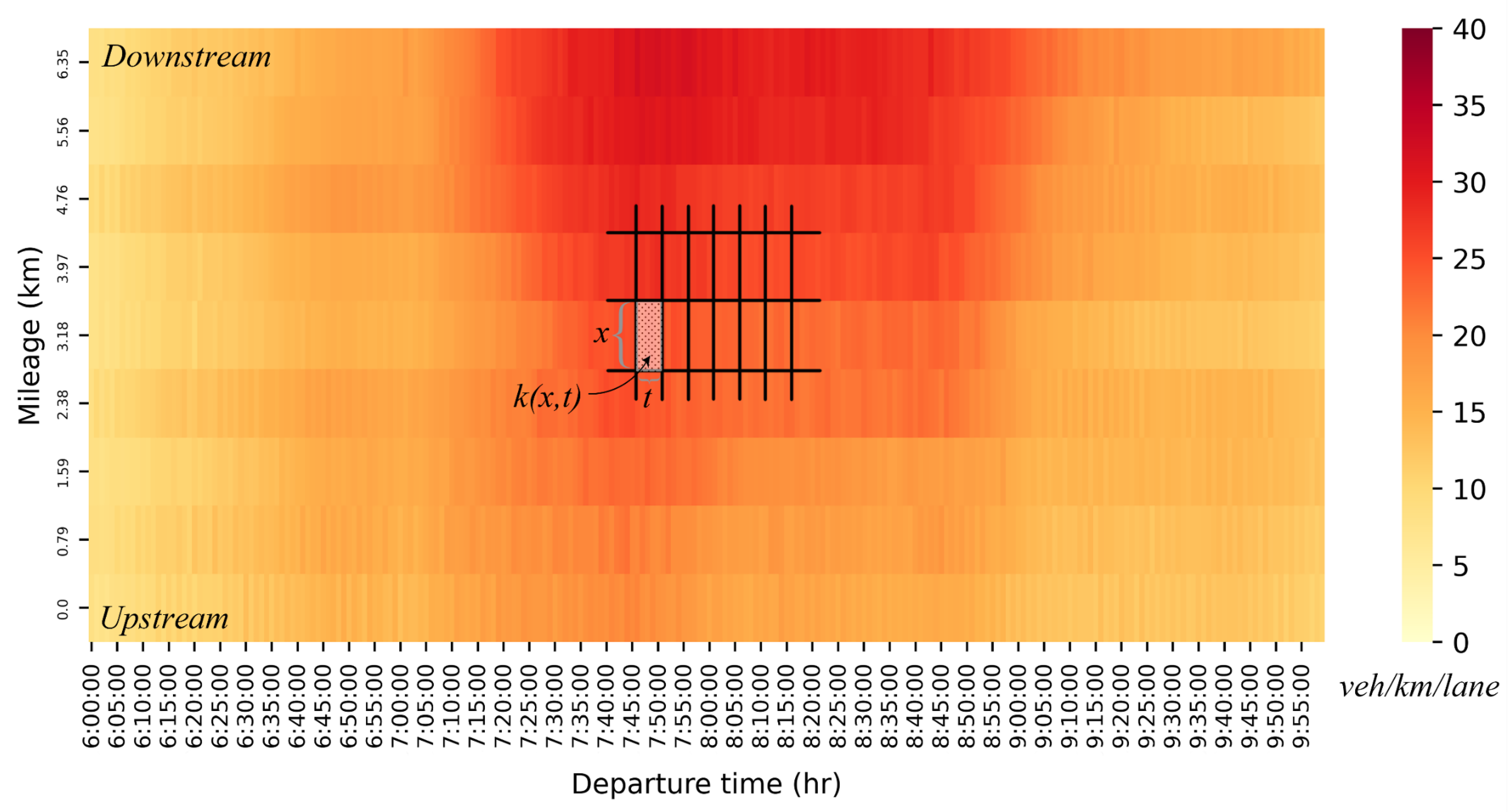
- For the parameters of the queueing model of the two selected freeway stretches, we calibrate the γ and ξ based on the **space-time density map** obtained from real data.

- According to minimizing the RSS of estimation of the density values of all space-time pixels:

$$\gamma^*, \xi^* = \arg\min_{\gamma,\xi}\left(\sum_{t=1}^{T}\sum_{x=1}^{X}(k(x,t) - \hat{k}(x,t))^2\right)$$

$$\text{s.t.} \quad \begin{cases} l - l_1(t) - l_2(t) > 0 \\ 0 < \gamma < 1 \end{cases}$$

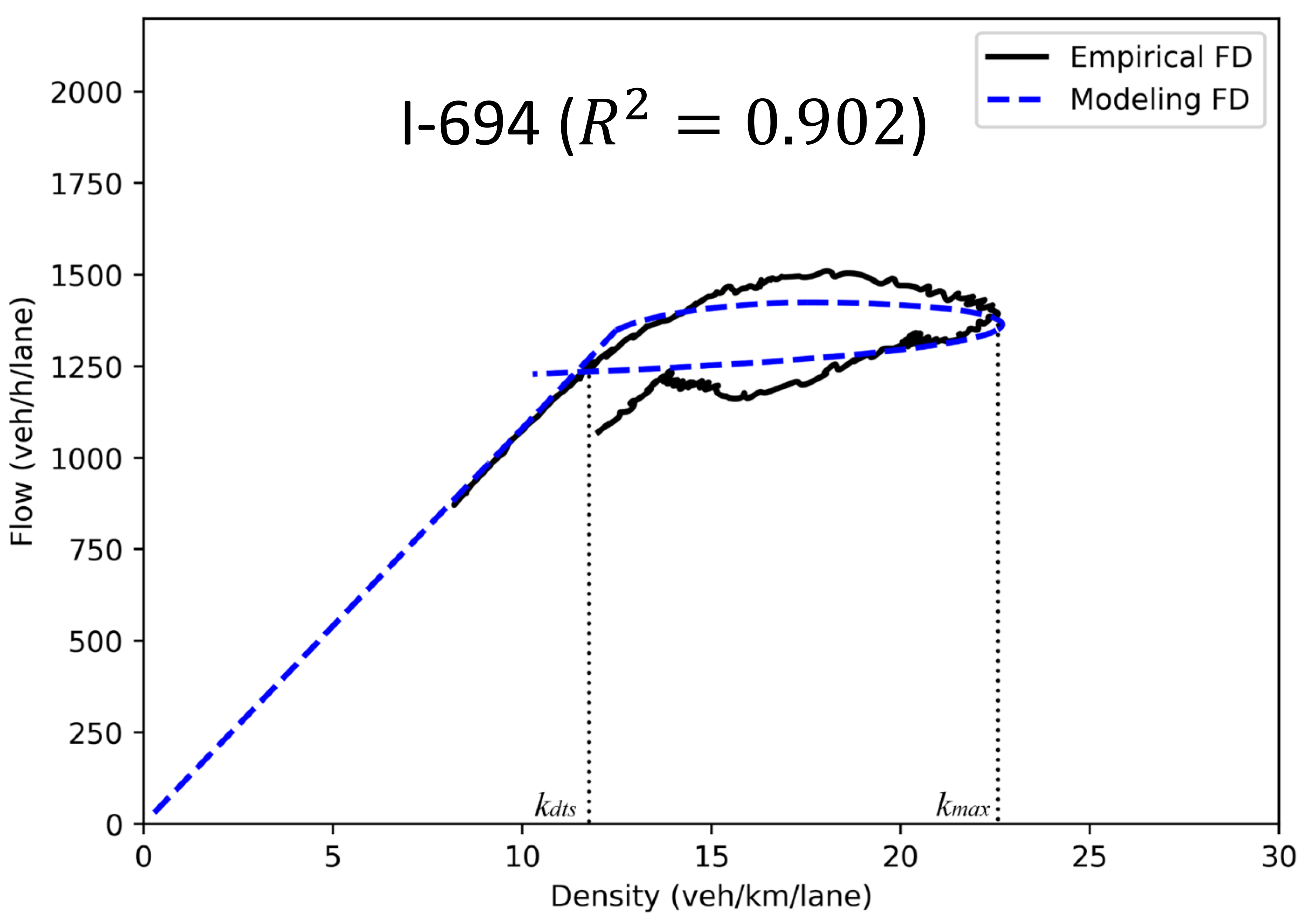
Where $k(x,t)$ and $\hat{k}(x,t)$ are the estimated and **real density value** at location x and time interval t obtained from the model and real data, respectively.



Model parameters calibration

- The **parameters calibration results** of the **arrival and departure flow models** and **queueing model** of the freeway stretches I-694 and I-805 are :

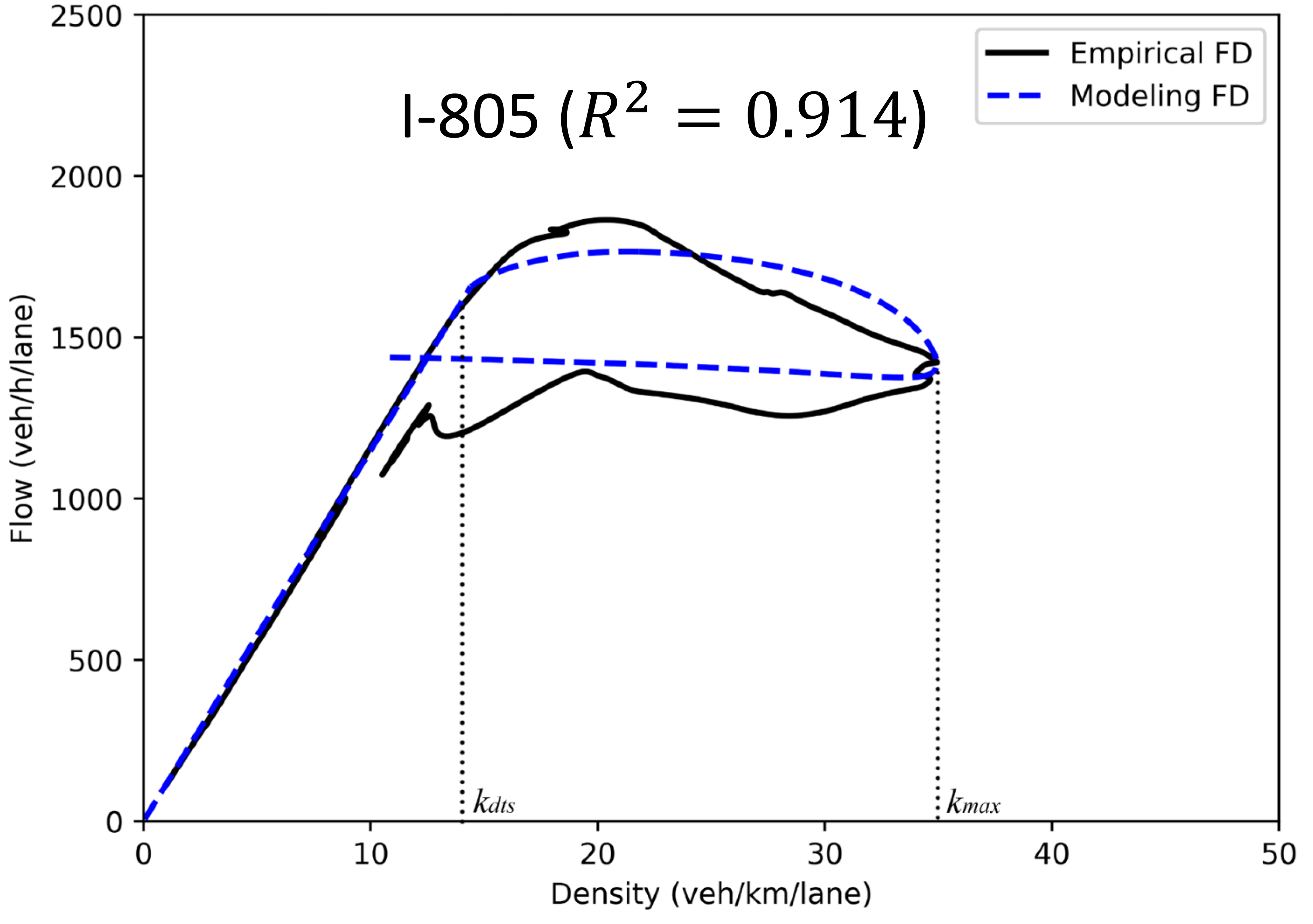
	I-694		I-805	
Parameters	Results	Std. Error	Results	Std. Error
Traffic flow model				
θ_a	0.0220	4.27e-4	0.0342	3.62e-4
θ_d	-	-	0.0351	2.78e-4
q_{drop}	-	-	241.788	18.549
α	0.856	-	0.813	-
RMSE		102.545		137.682
R^2		0.917		0.924
Queueing model				
γ	0.522	3.62e-3	0.362	2.87e-3
ξ	0.268	1.84e-3	0.398	3.24e-3
RMSE		3.168		4.971
R^2		0.904		0.931



Estimated based on ξ and γ:

$k_{dts} = 12.46\ veh/km/lane$

$k_{max} = 23.87\ veh/km/lane$



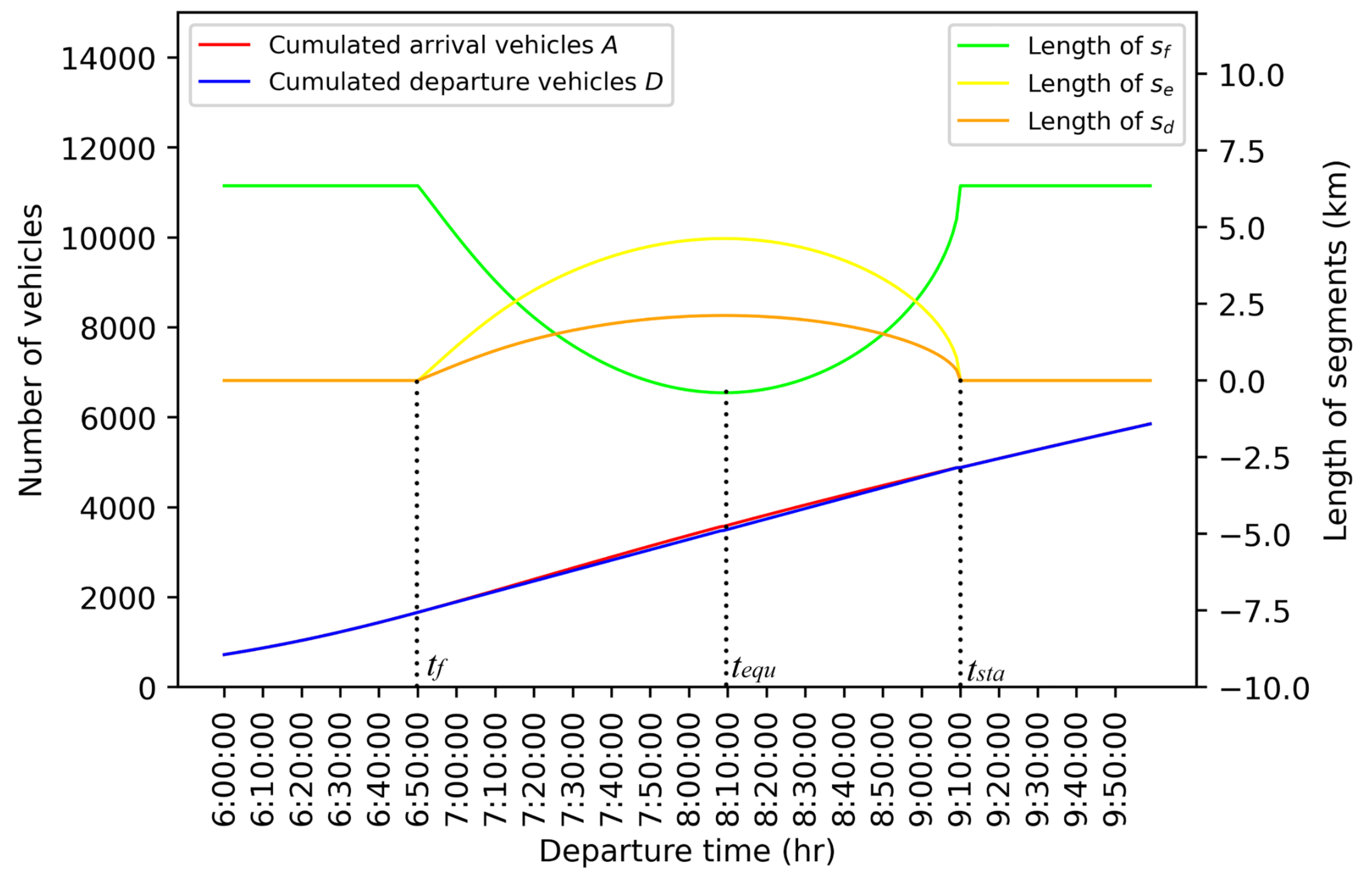
Estimated based on ξ and γ:

$k_{dts} = 13.29\ veh/km/lane$

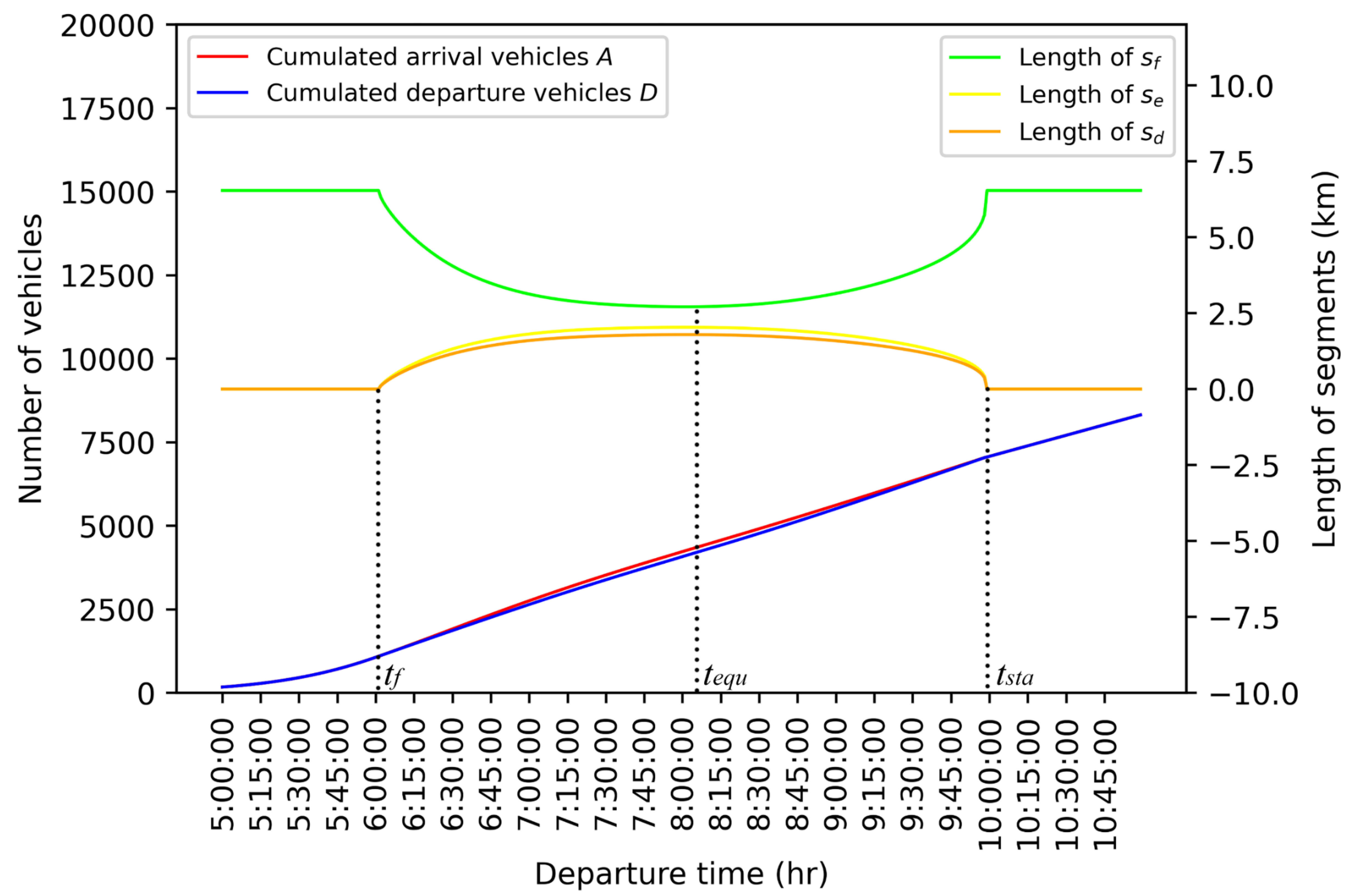
$k_{max} = 36.71\ veh/km/lane$

Model parameters calibration

- The change in **cumulative arrival** and **departure vehicles** and **the length of the different segments** over time of the selected freeway stretches I-694 and I-805:



I-694

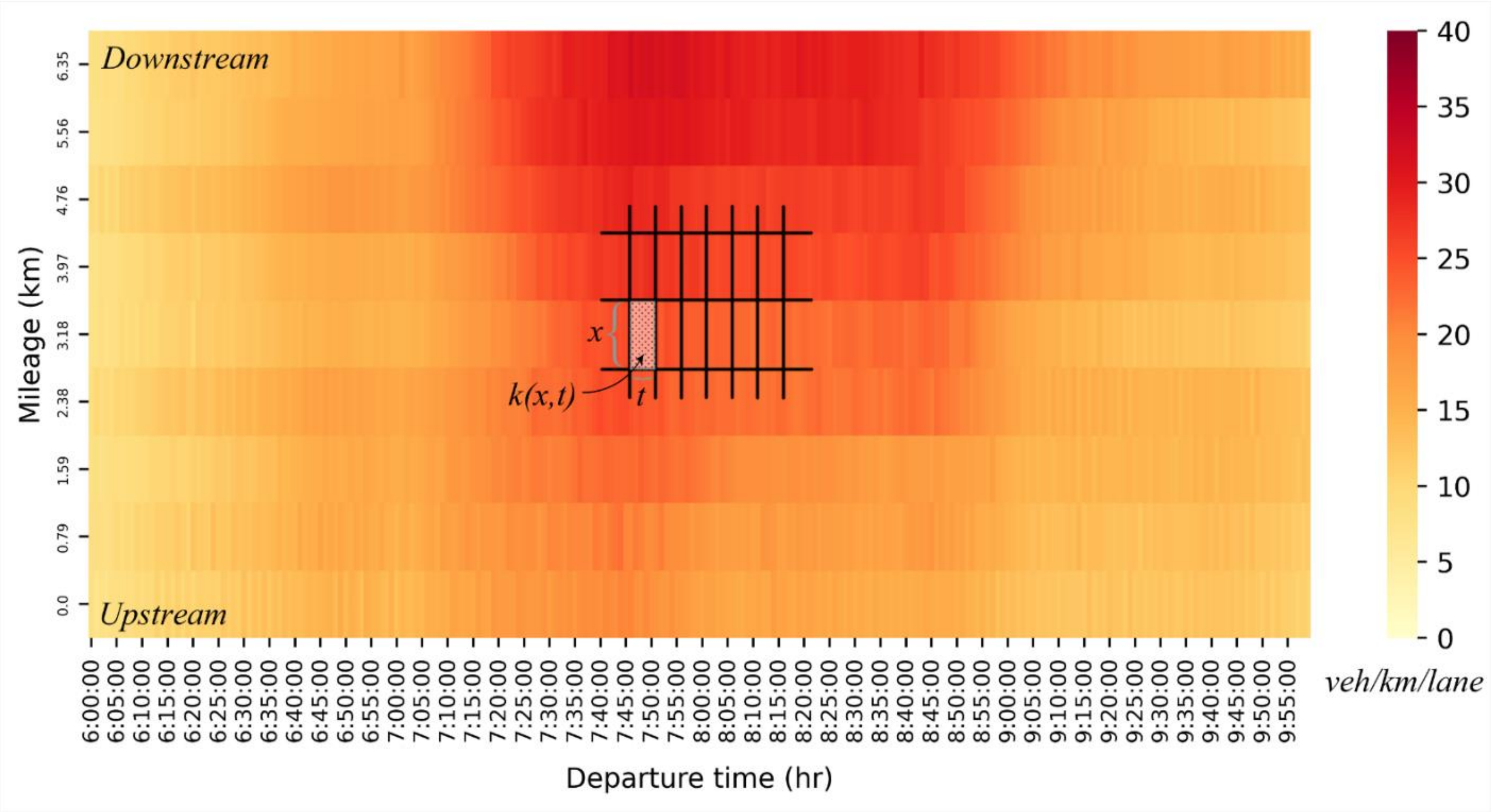


I-805

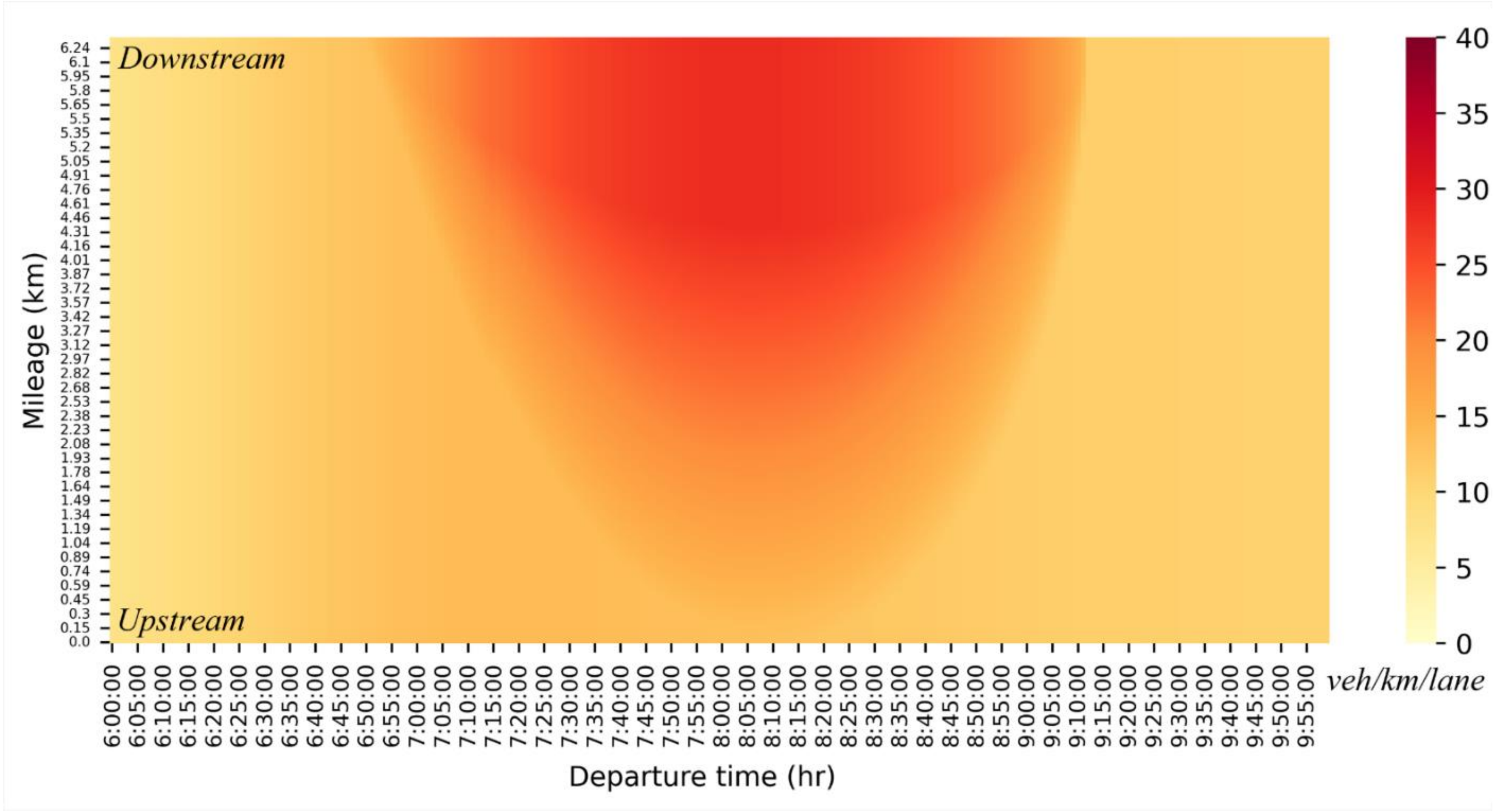
Model parameters calibration

- The **density space-time maps** based on real data and proposed model of the freeway stretch I-694 (W-E):

real data

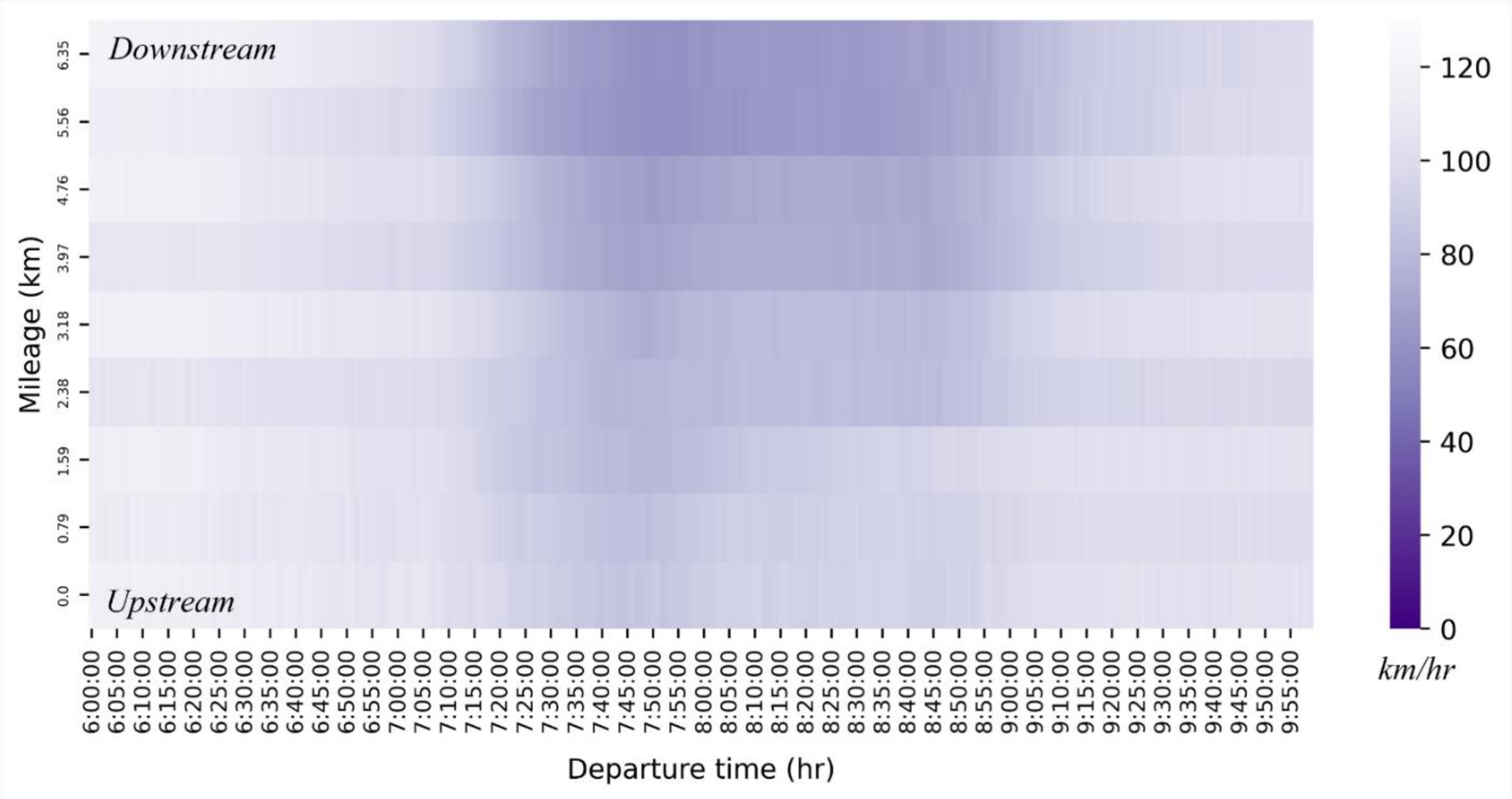


proposed model

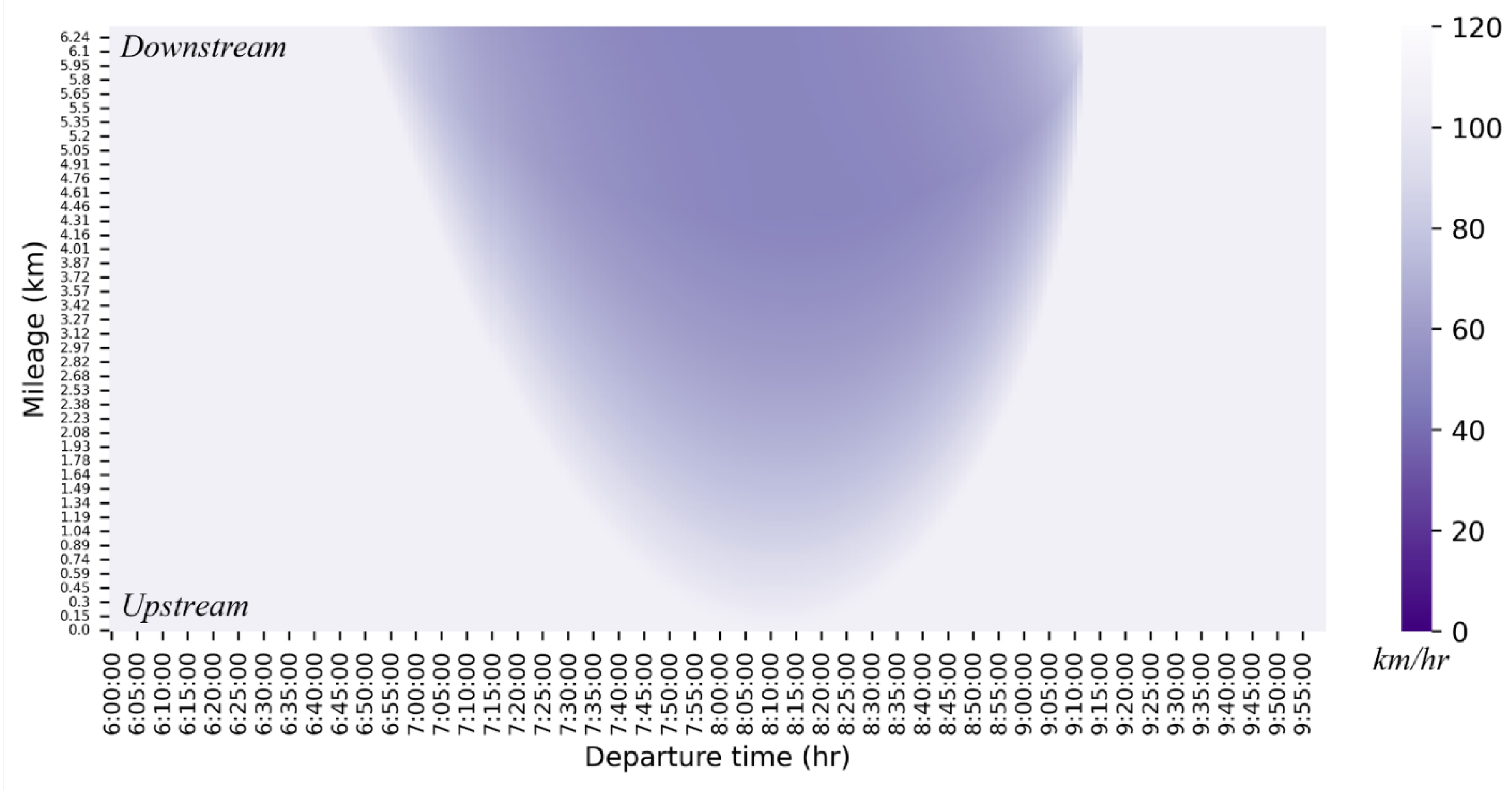


- The **speed space-time maps** based on real data and proposed model of the freeway stretch I-694 (W-E):

real data



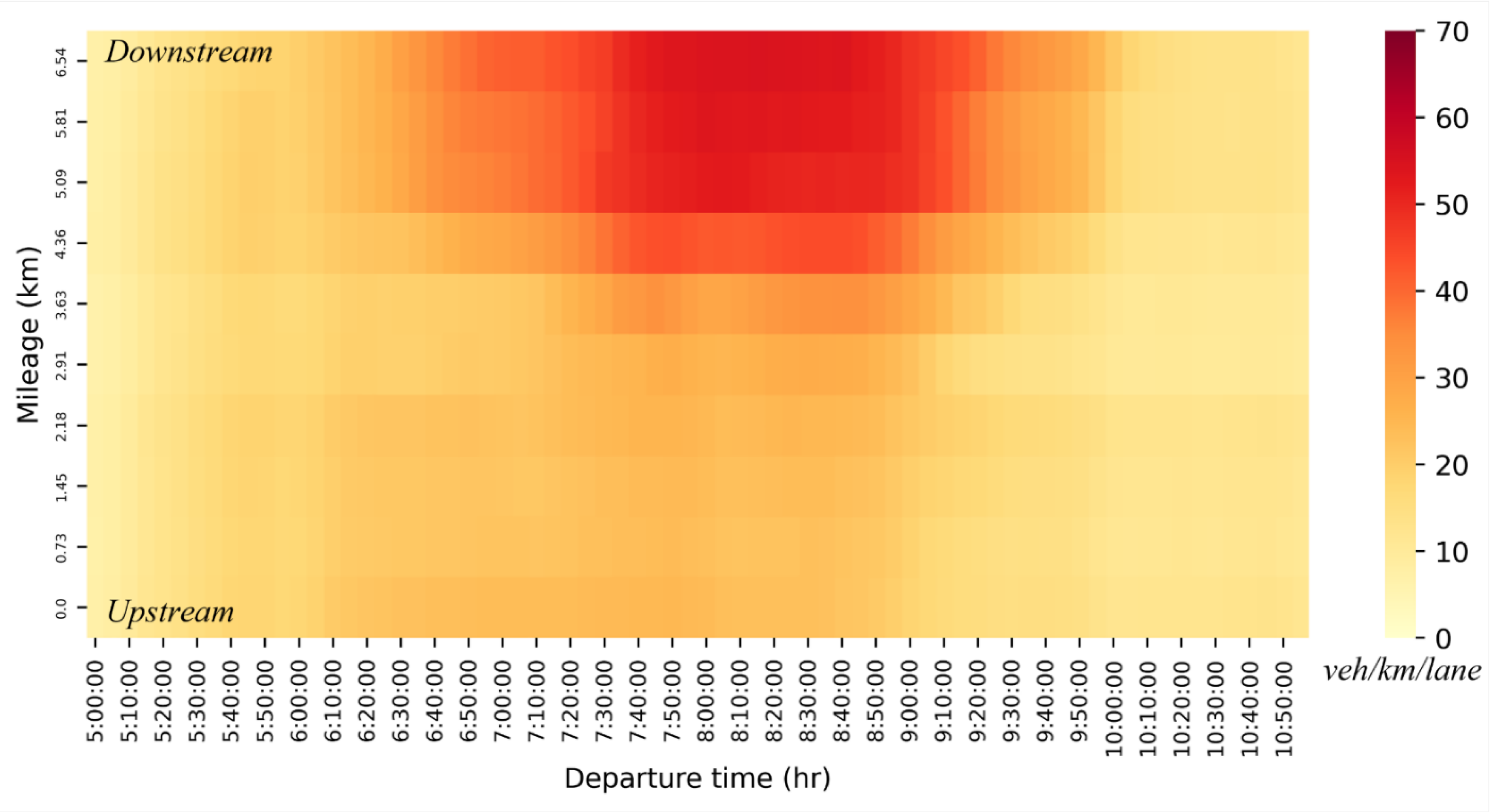
proposed model



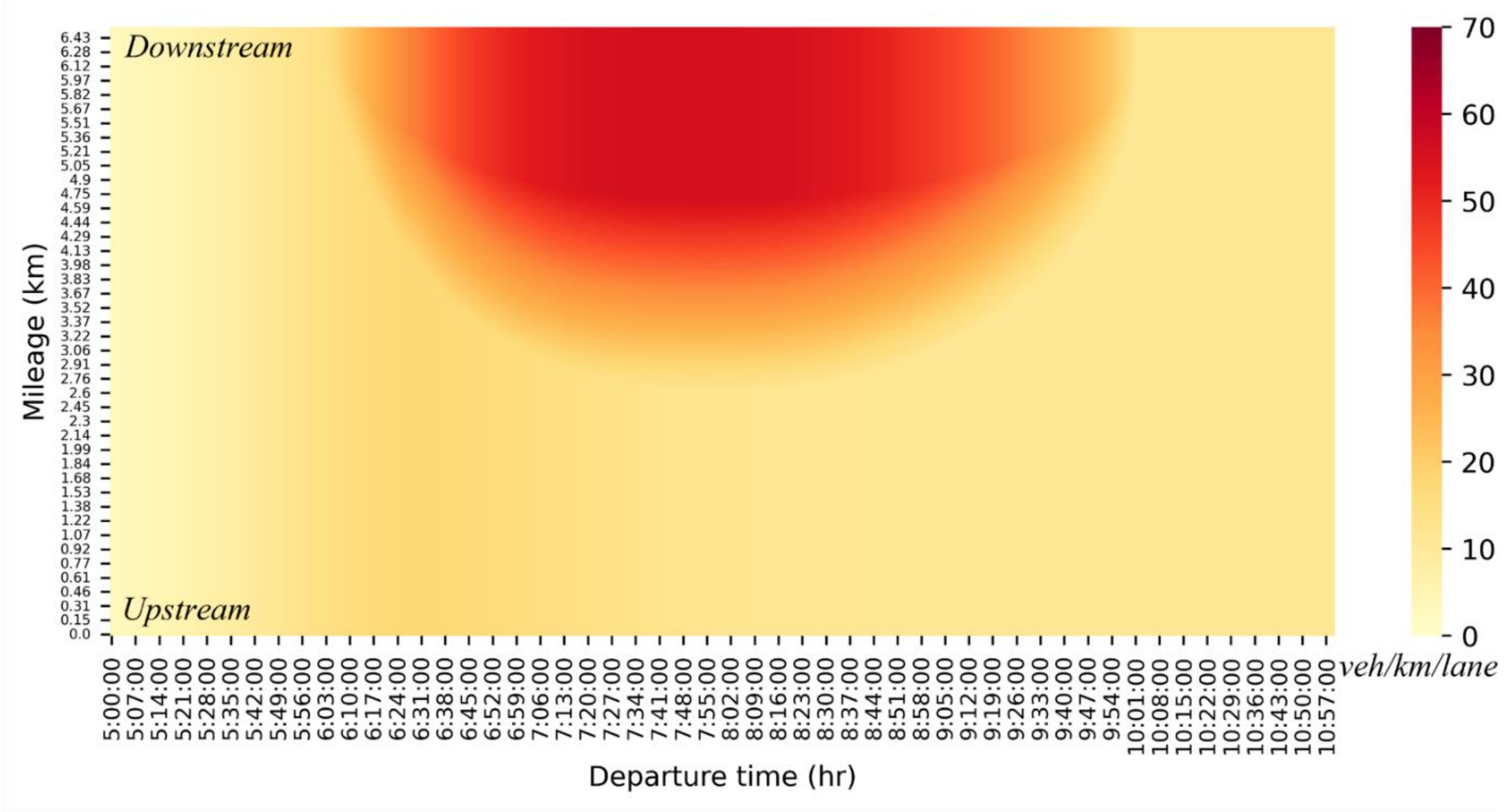
Model parameters calibration

- The **density space-time maps** based on real data and proposed model of the freeway stretch I-805 (S-N):

real data

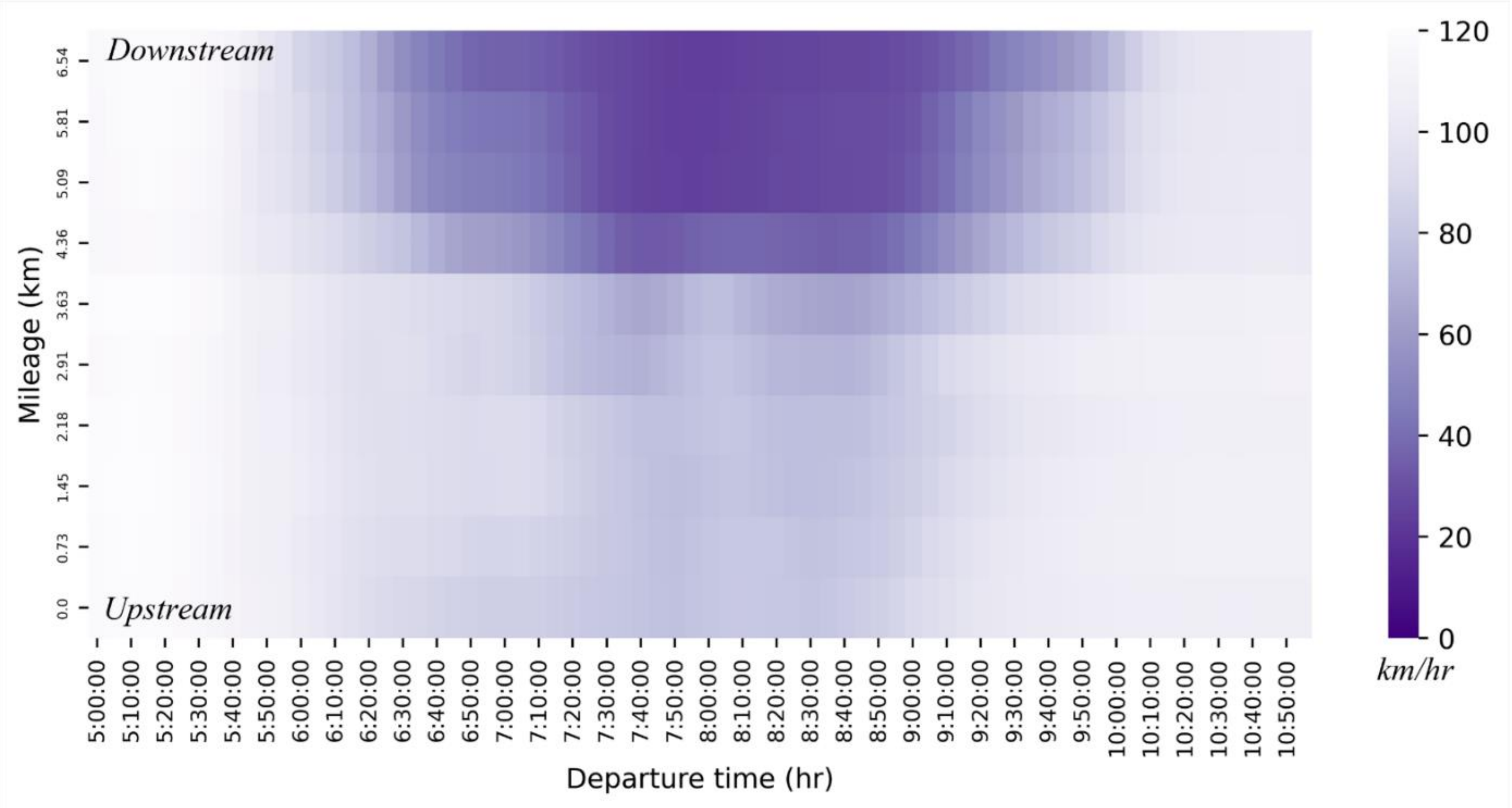


proposed model

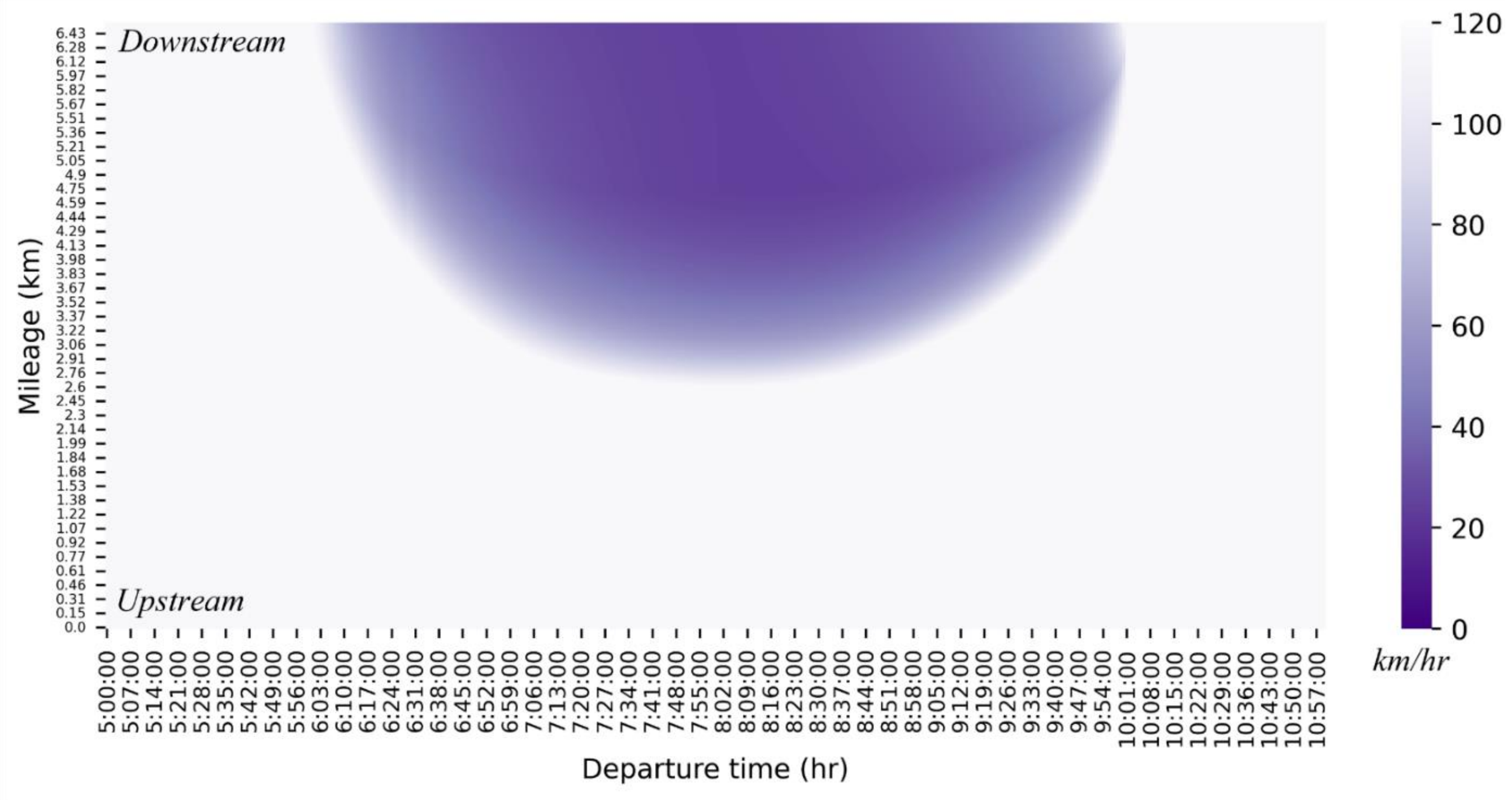


- The **speed space-time maps** based on real data and proposed model of the freeway stretch I-805 (S-N):

real data

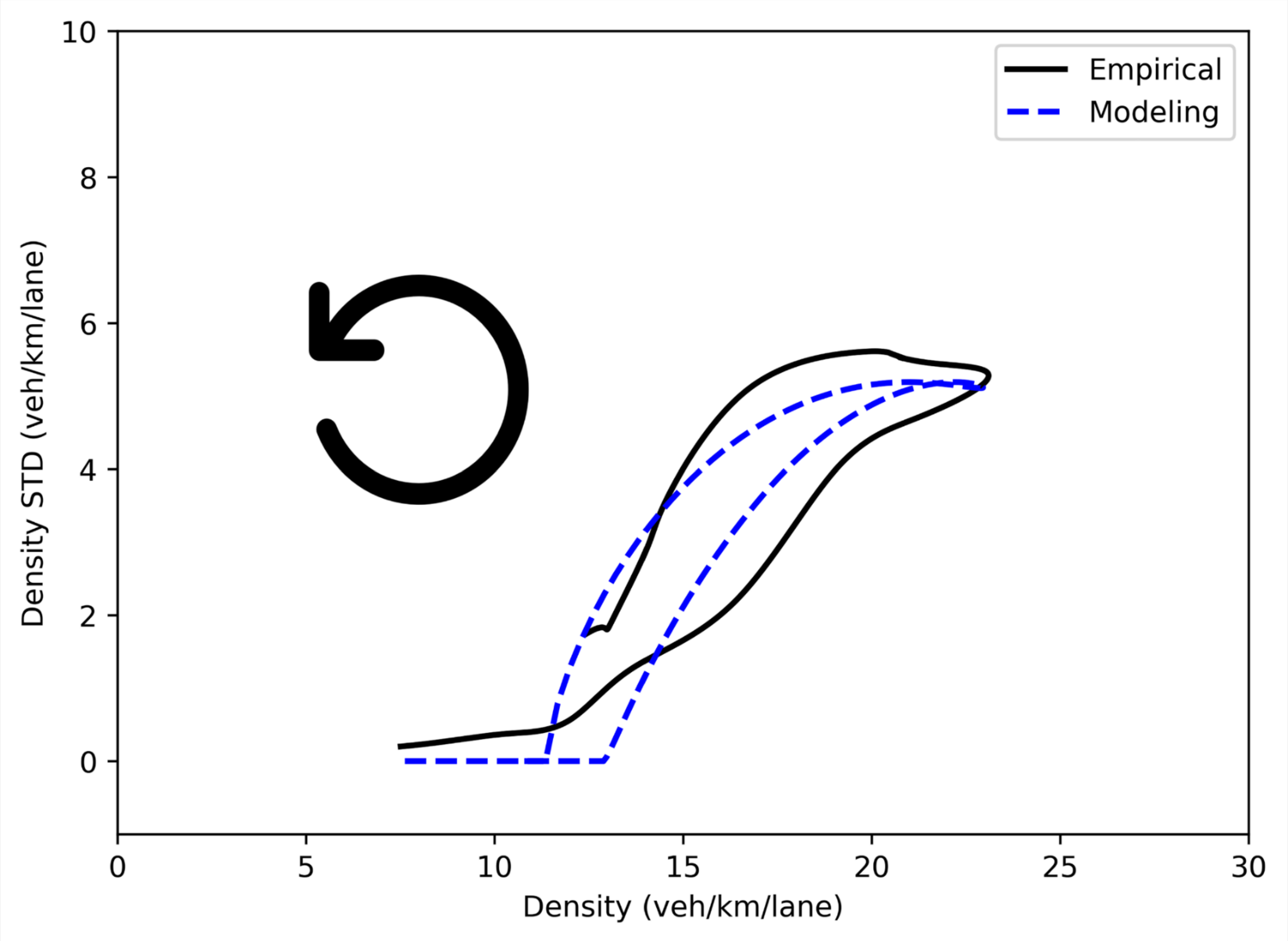


proposed model

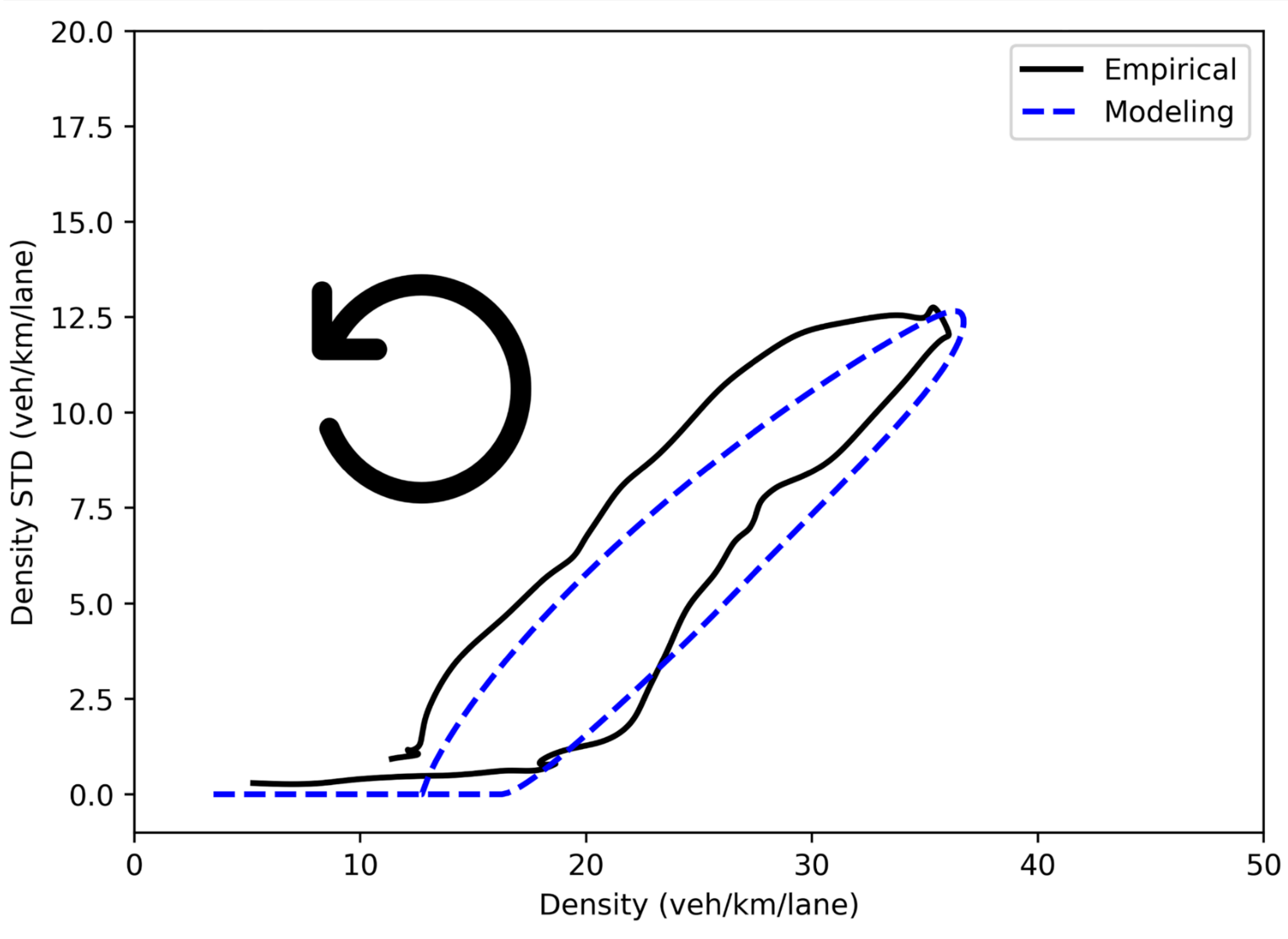


Model parameters calibration

- Furthermore, we validate the relationship between the average density and spatial density heterogeneity, here indicated as its **standard deviation (STD)**:



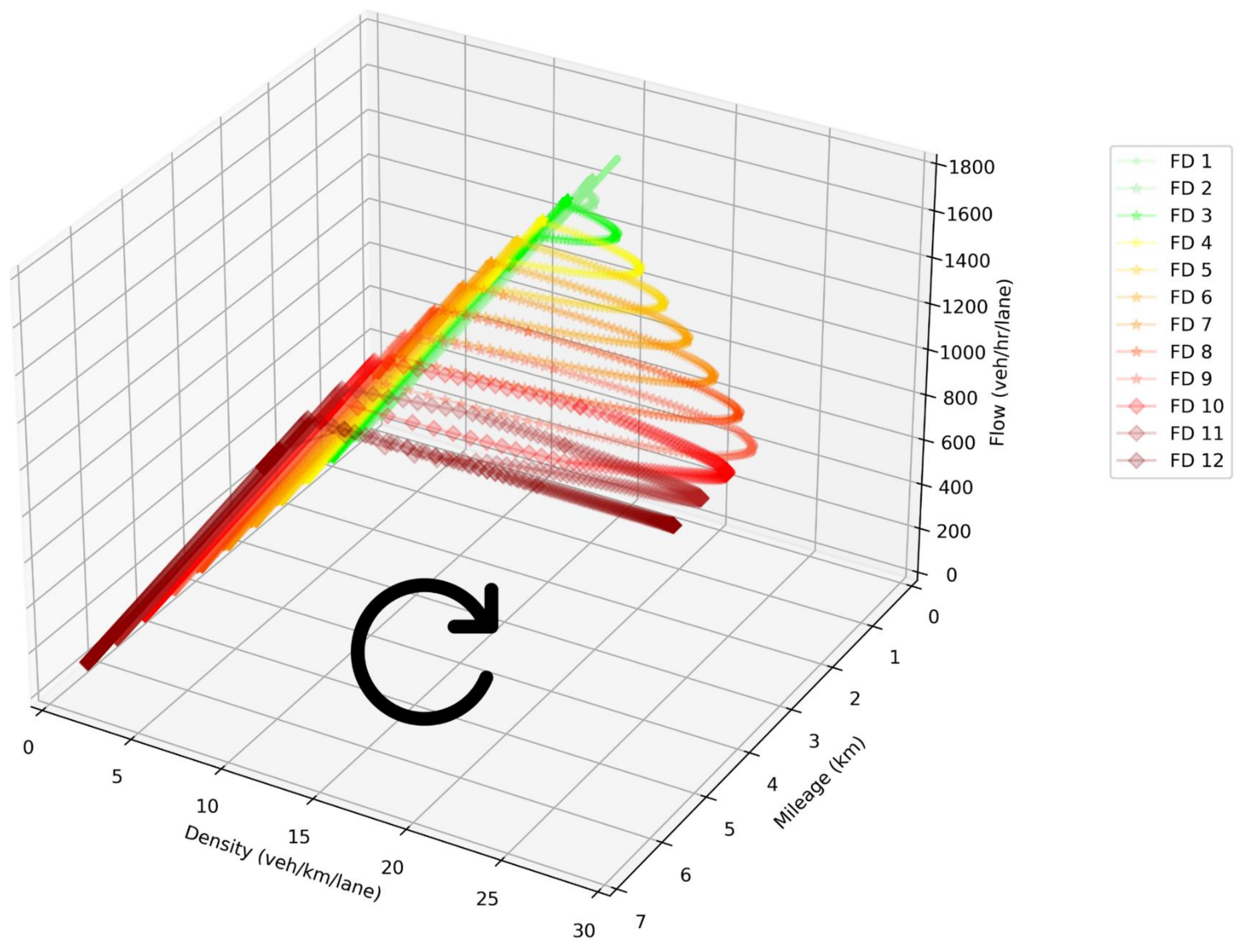
I-694 ($R^2 = 0.863$)



I-805 ($R^2 = 0.891$)

Model parameters calibration

- Based on the calibrated model, we obtain the **FDs** of **different spatial points** in the freeway stretch **I-694** :



The 3D-FDs of different spatial points in the freeway stretch I-694

Conclusions

- For the first time, we define the **arrival** and **departure flow functions** that conform to the **characteristics** of the **morning peak flow** based on the **logistic functions**, and it consistent with to the **hysteresis loop phenomenon** that appears in the fundamental diagrams (FDs).

- We **pioneer** a **multi-stage** description of **queueing** process, so as to realize the **continuity of flow and density in space** and consider the queue **expansion** and **spillback** scenarios.

- The calibrated models are **validated** in various ways, and the results are all **consistent** with the **actual scenarios**.

References

1. Lu, J., C. Li, X. B. Wu, and X. S. Zhou (2023). Physics-informed neural networks for integrated traffic state and queue profile estimation: A differentiable programming approach on layered computational graphs. Transportation Research Part C: Emerging Technologies 153, 104224.

2. Newell, G. F. (1988). Traffic flow for the morning commute. Transportation science 22 (1), 47–58.

3. Seo, T., A. M. Bayen, T. Kusakabe, and Y. Asakura (2017). Traffic state estimation on highway: A comprehensive survey. Annual reviews in control 43, 128–151.

4. Wang, Y. and M. Papageorgiou (2005). Real-time freeway traffic state estimation based on extended Kalman filter: a general approach. Transportation Research Part B: Methodological 39 (2), 141–167.

5. Zhou, X. S., Q. Cheng, X. Wu, P. Li, B. Belezamo, J. Lu, and M. Abbasi (2022). A meso-to-macro cross-resolution performance approach for connecting polynomial arrival queue model to volume-delay function with inflow demand-to-capacity ratio. Multimodal Transportation 1 (2), 100017.

Thanks!

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