

Game-theoretic Modelling of Integrated Longitudinal and Lateral Vehicle Maneuvers

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Research Overview

Objectives:

1. Model microscopic vehicle behaviour in a realistic manner.
2. Capture the decision-making logic of human drivers.
3. Jointly considering car-following and lane-changing maneuvers.
4. Focus on discretionary lane change only.

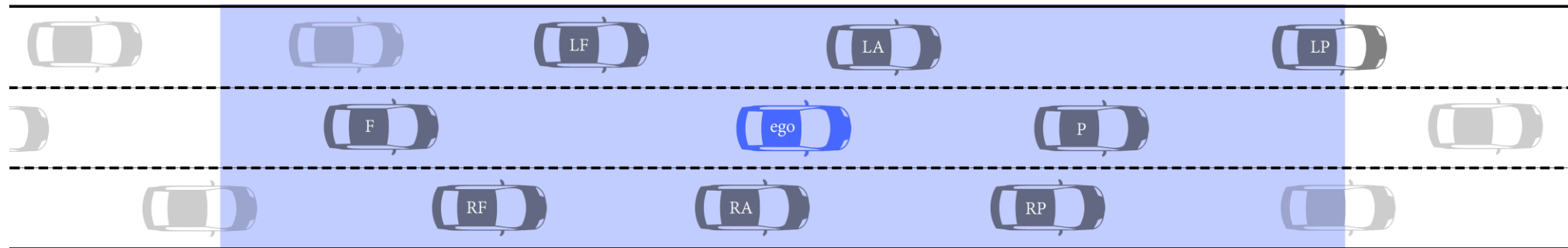
Research Overview

Assumptions:

1. **Rationality:** Drivers are rational.
2. **Predictability:** Drivers can foresee a short time interval into the future and will use this prediction to make optimal decisions.
3. **Heterogeneity:** Each human driver has his/her own preference on the weighting of their costs.

Identify Game Opponents

- A set of N players
- At most 8 players can be identified

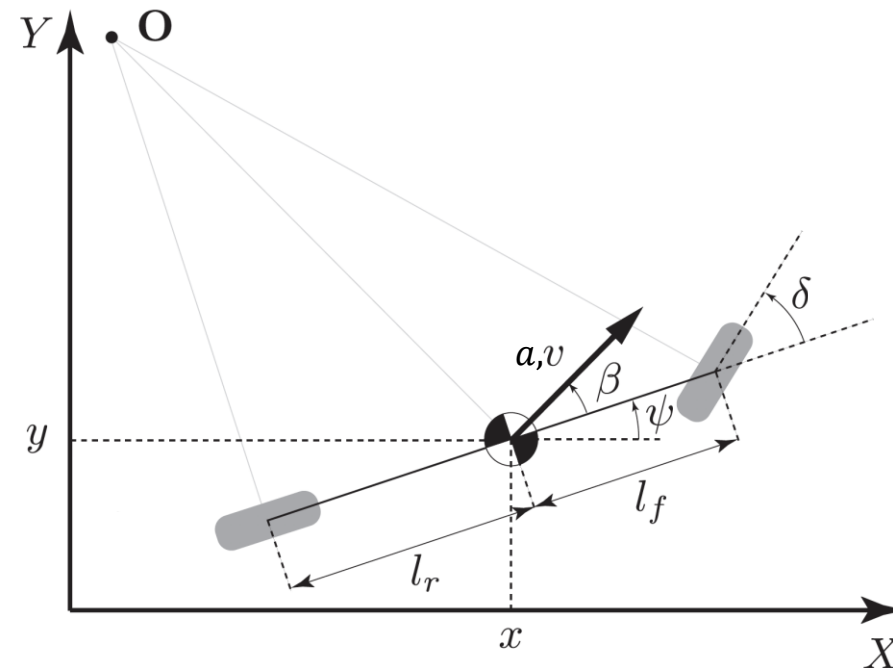


Rectangle boundary (L by 3W)

System Dynamics

Kinematic Bicycle Model:

- States: x-position, y-position, velocity, heading ($\mathbf{z} = [x, y, v, \psi]$)
- Controls: acceleration, steering angle ($\mathbf{u} = [a, \delta]$)
- State dynamics based on $\mathbf{z}(k + 1) = \mathbf{f}(\mathbf{z}(k), \mathbf{u}(k)) + \mathbf{z}(k)$



Receding Horizon Cost Optimization

The optimal control \mathbf{u}^* is derived such that the cost function J is minimized during the planning period $[t_0, t_f)$ subject to the state dynamics and the initial condition.

$$\mathbf{u}_{[t_0, t_f)}^* = \arg \min_{\mathbf{u}} J(\mathbf{z}, \mathbf{u}, t | \mathbf{z}(t_0))$$

$$J(\mathbf{z}, \mathbf{u}, t | \mathbf{z}(t_0)) = \sum_{t=t_0}^{t_f} \eta^{t-t_0} \mathcal{L}(\mathbf{z}(t), \mathbf{u}(t), t)$$

The running cost \mathcal{L} :

$$\mathcal{L}(\mathbf{z}, \mathbf{u}, t) = \sum_k \alpha_k \mathcal{L}_k(\mathbf{z}, \mathbf{u}, t)$$

Cost Function – Speed

The car following behaviour is modelled according to the Intelligent driving model (IDM). The cost function then minimizes the discrepancy between the actual speed and the equilibrium speed.

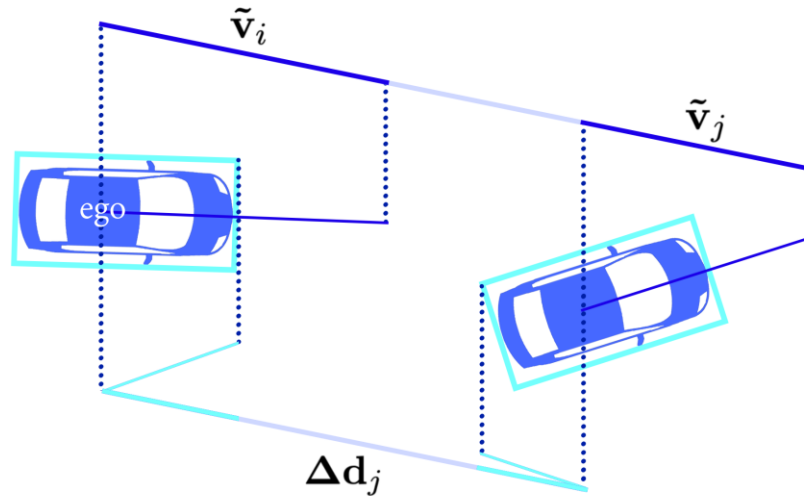
$$\mathcal{L}_{speed} = (v_{eq} - v_i)^2$$

$$v_{eq} = \min \left\{ v_d, \frac{s - s_0}{T} \right\}$$

Cost Function – Safety

The safety cost is applied to all surrounding opponents in the set N which penalizes low time to collision (TTC) between the vehicles.

$$\mathcal{L}_{safety} = \sum_{j \in N} \exp(-TTC_j)$$



Cost Function – Comfort

To optimize comfort, we penalize excessive control inputs.

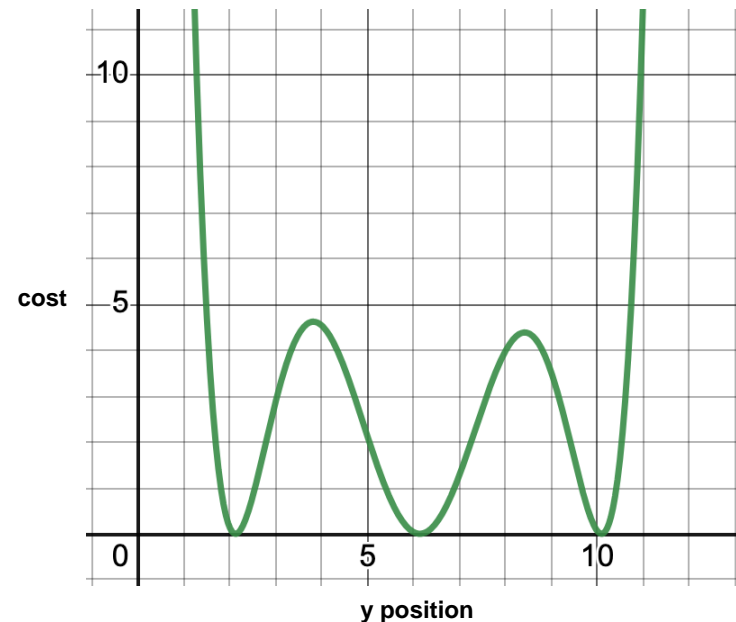
$$\mathcal{L}_{acceleration} = a_i^2$$

$$\mathcal{L}_{steering} = \delta_i^2$$

Cost Function – Centring

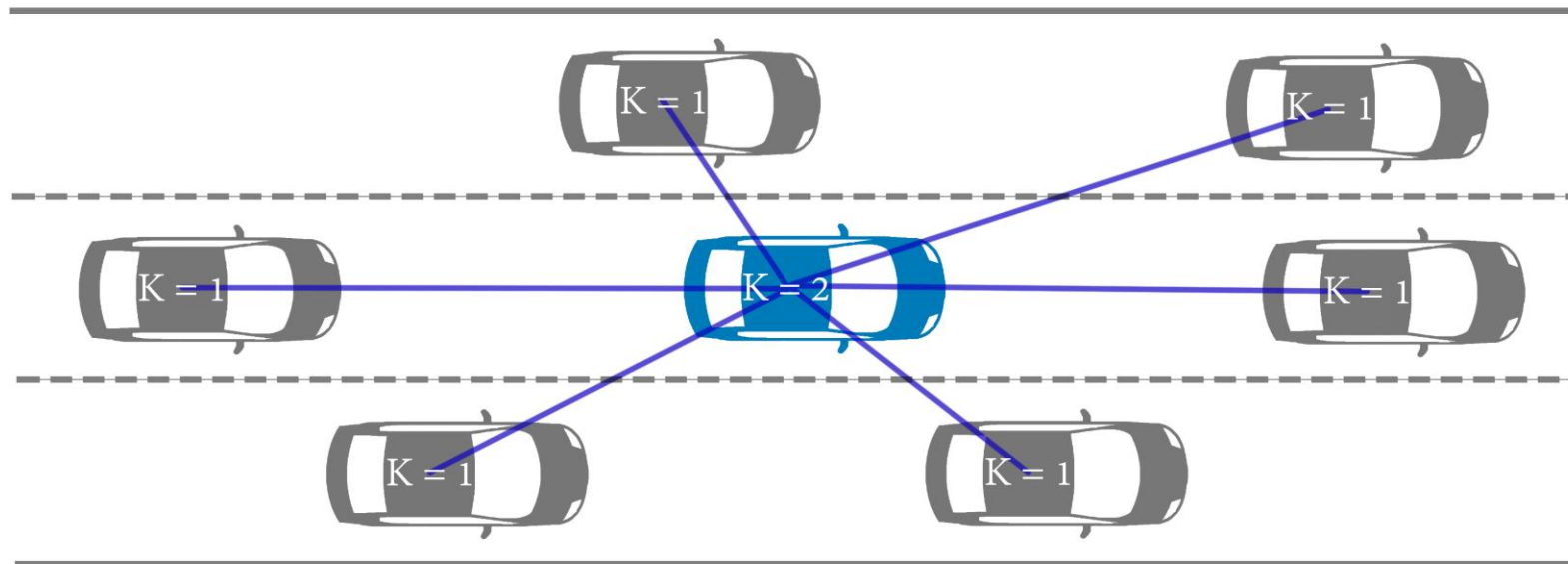
The centring cost penalizes the vehicle if it deviates from the centre of the lane in which it is classified to be in. It can be thought of as a lane keeping cost.

$$\mathcal{L}_{centre} = \prod_{c \in L} \frac{(y_i - c)^2}{c}$$



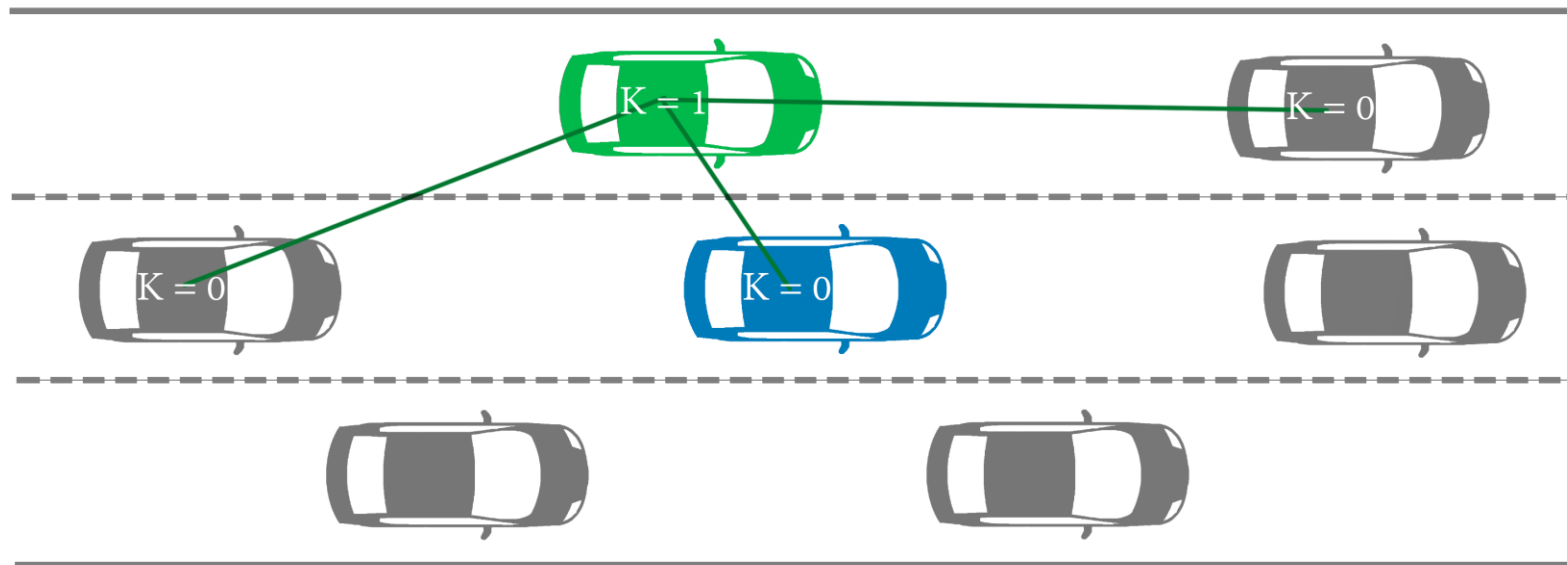
Level-K Game Theory

- Complete information level-k game.
- Vehicle with level K assumes all its opponents have levels $K-1$ and will act accordingly.



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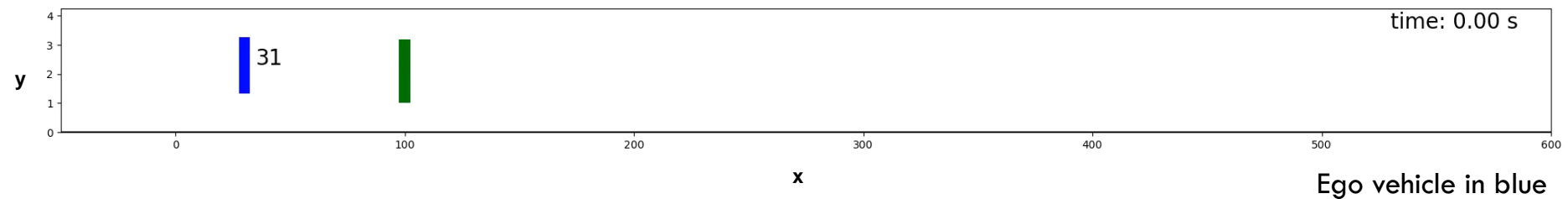
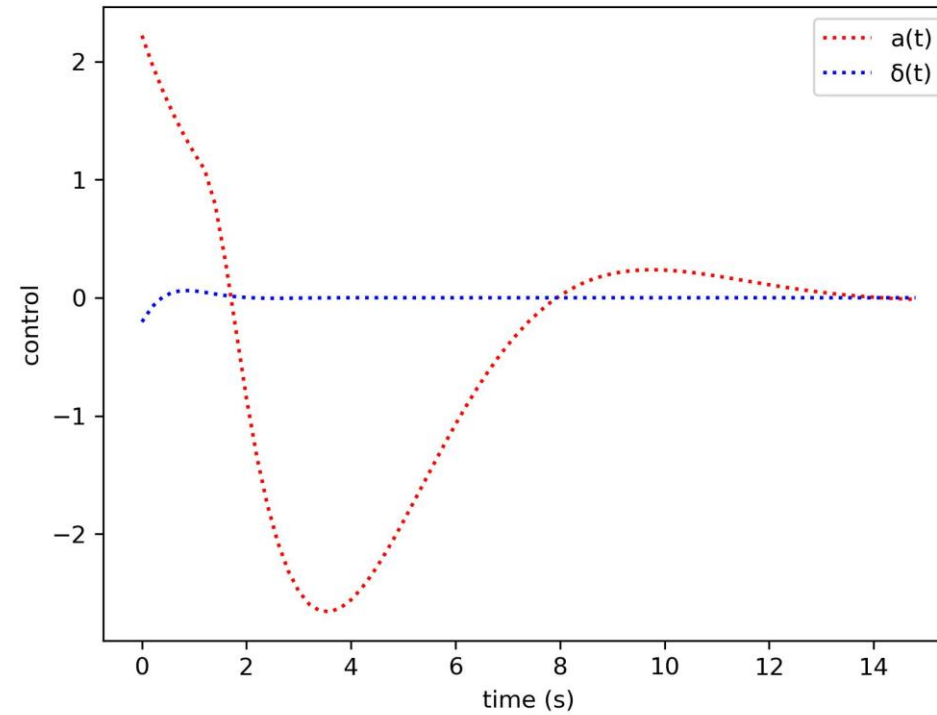
Test scenario – 1 lane

- Global optimization
- 0.2 s interval, 15 s duration
- 5 step prediction horizon, 1 step control horizon
- Computation time: 1.02 s

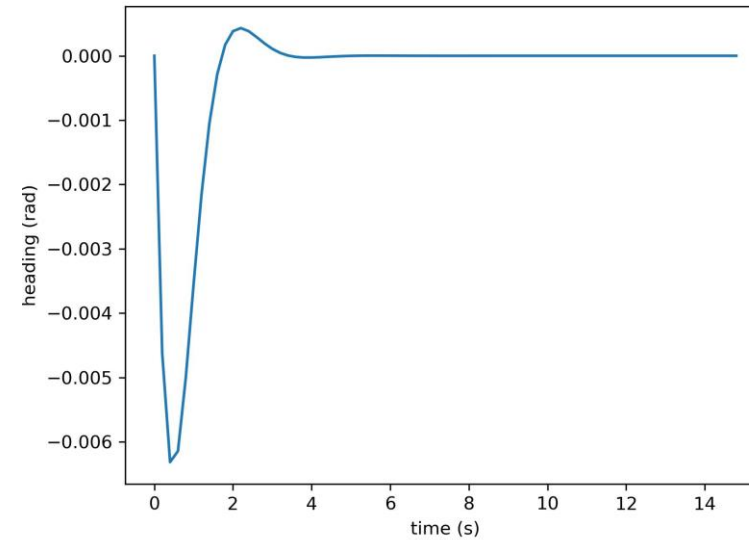
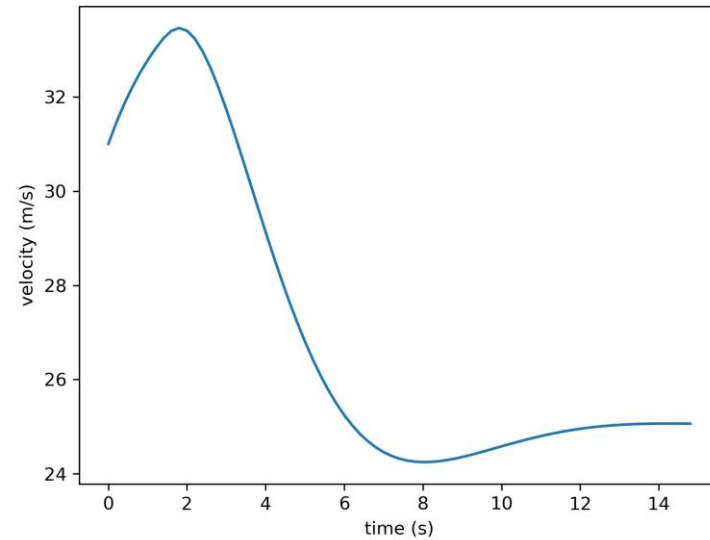
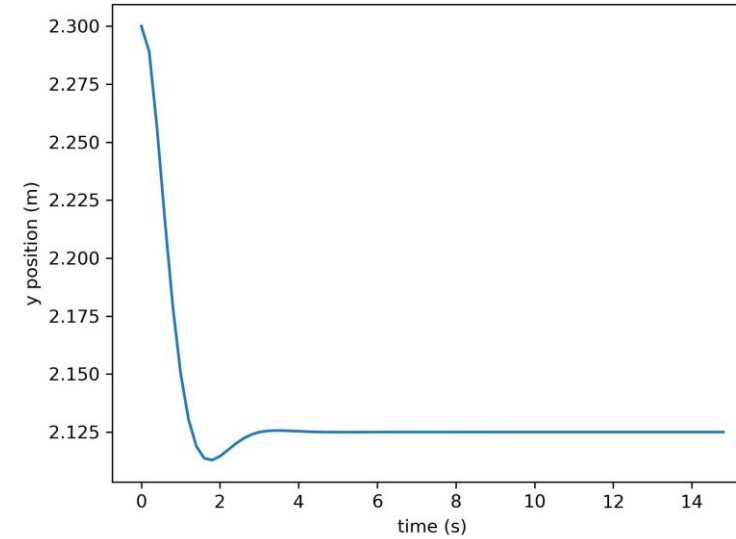
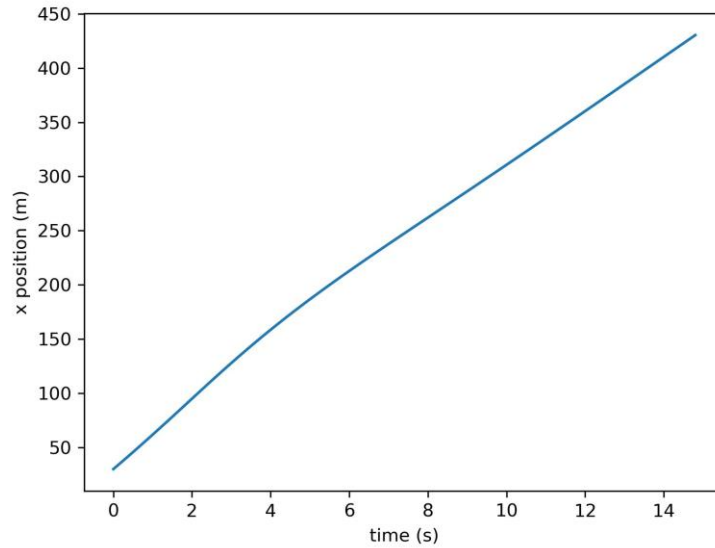
- Discount factor: 1
- Desired speed: 35 m/s
- Ego vehicle weights:

Speed	Safety	acceleration	steering	Lane centre
0.1	0.3	0.006	0.001	0.01

Test scenario – 1 lane



Test scenario – 1 lane



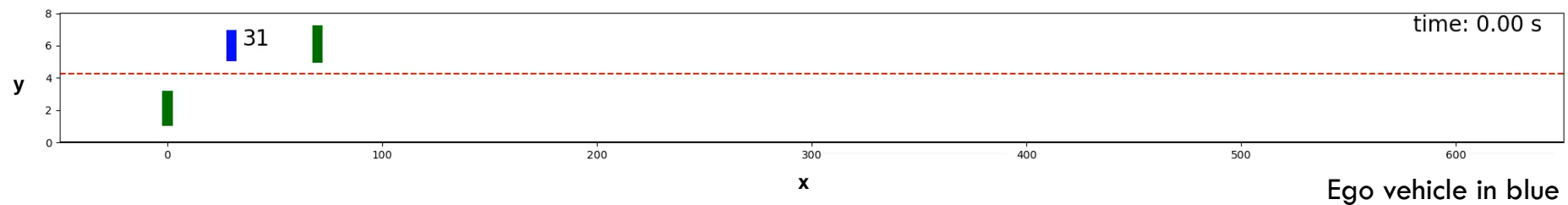
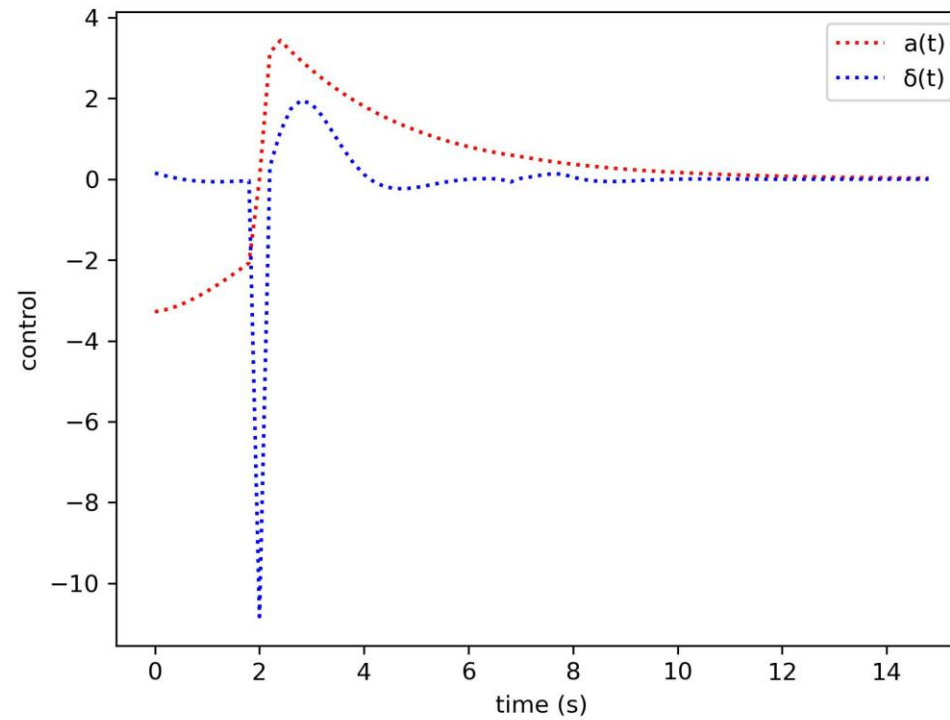
Test scenario – 2 lanes

- Global optimization
- 0.2 interval, 15 second duration
- 5 step prediction horizon, 1 step control horizon
- Computation time: 1.35 s

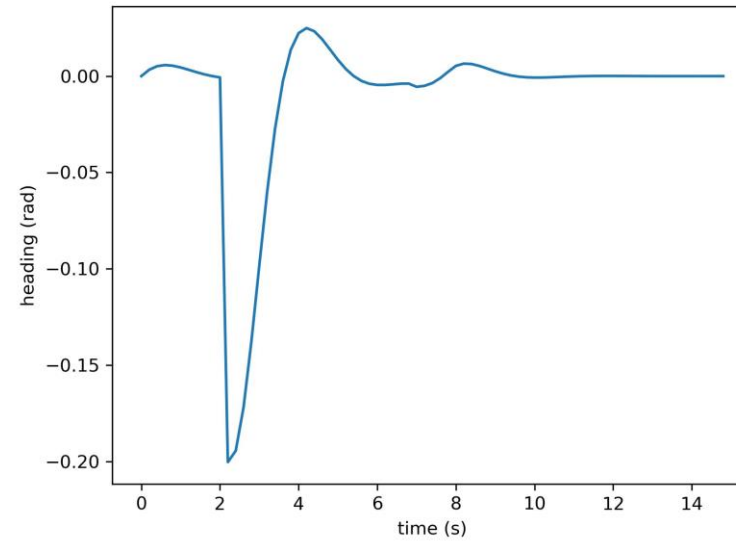
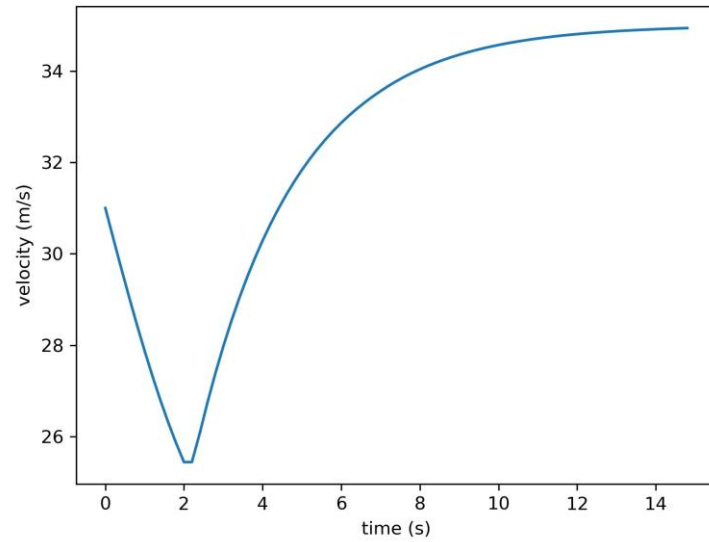
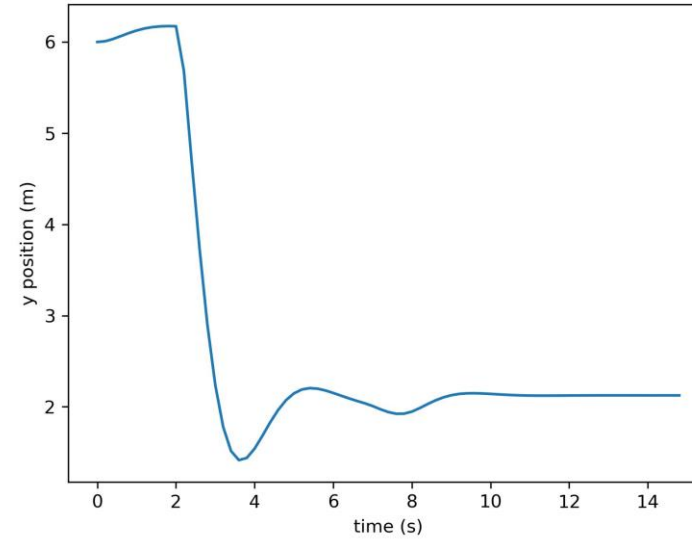
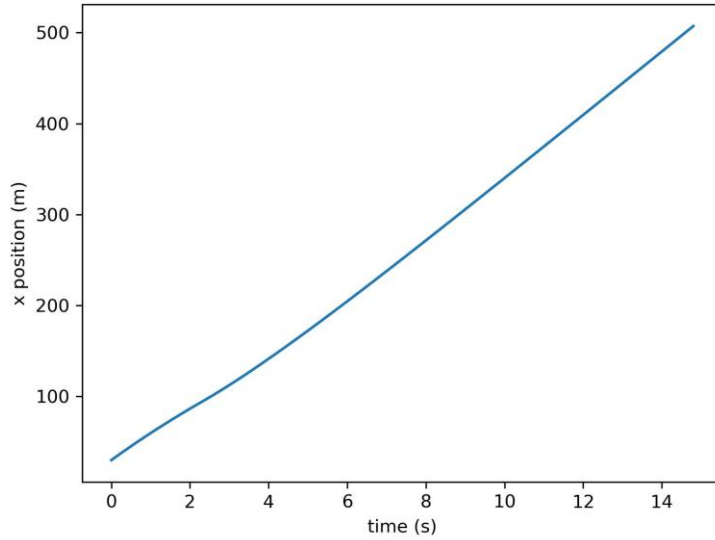
- Discount factor: 1
- Desired speed: 35 m/s
- Ego vehicle weights:

Speed	Safety	acceleration	steering	Lane centre
0.06	0.1	0.06	0.01	0.05

Test scenario – 2 lanes



Test scenario – 2 lanes



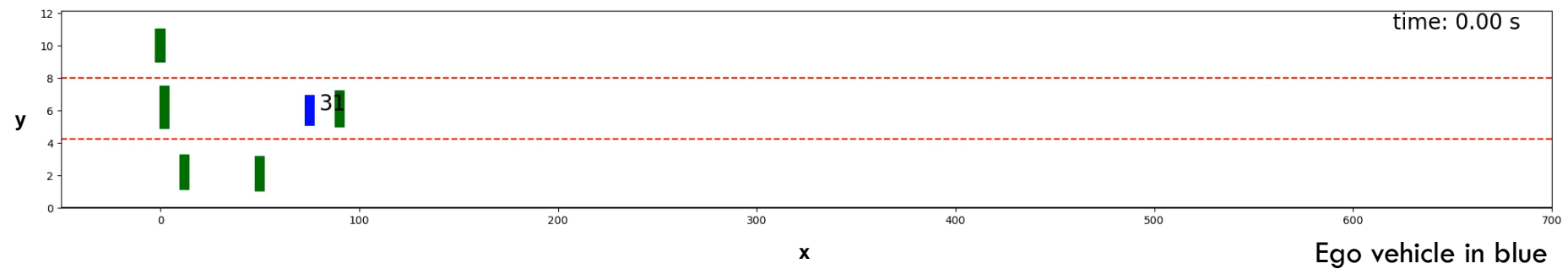
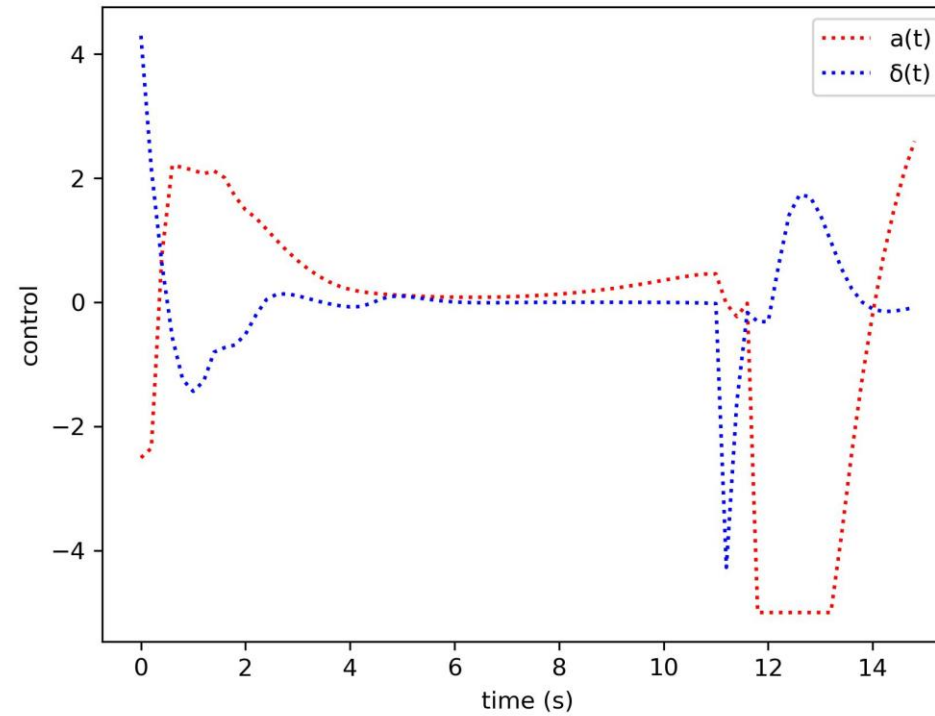
Test scenario – 3 lanes

- Global optimization
- 0.2 interval, 15 second duration
- 5 step prediction horizon, 1 step control horizon
- Computation time: 6.00 s

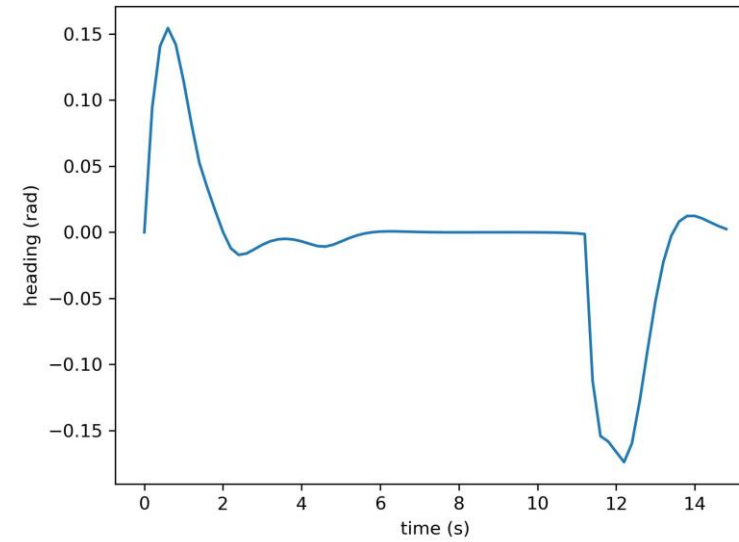
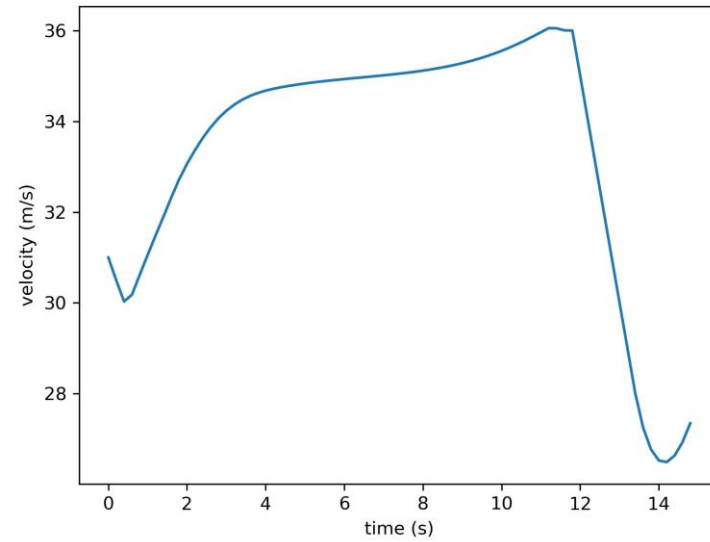
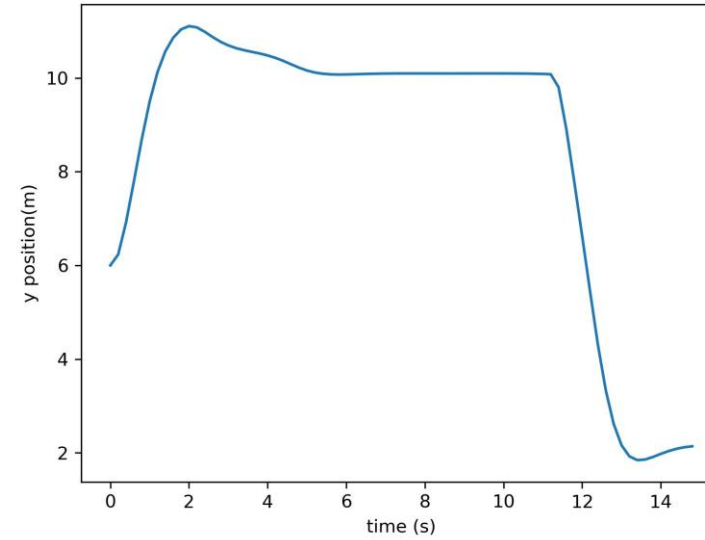
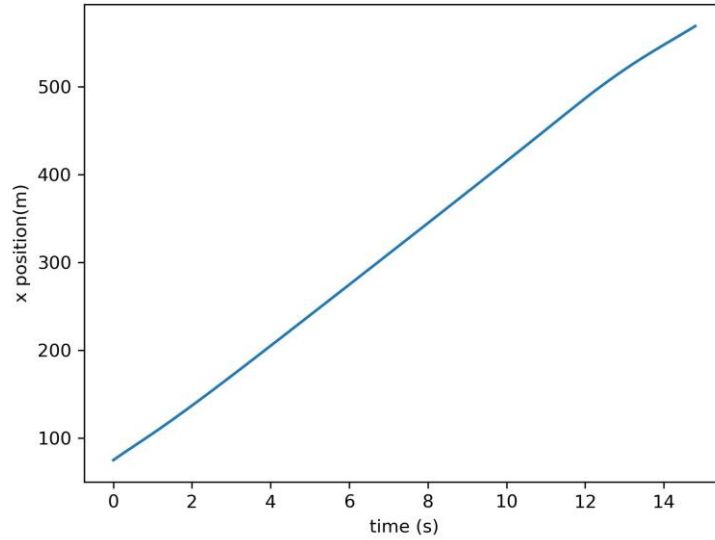
- Discount factor: 1
- Desired speed: 35 m/s
- Ego vehicle weights:

Speed	Safety	acceleration	steering	Lane centre
0.01	0.3	0.006	0.001	0.006

Test scenario – 3 lanes



Test scenario – 3 lanes



Model Calibration

HighD dataset:

- Vehicle trajectory over a highway segment ($\sim 400\text{m}$)
- Vehicle data $(x, y, v_x, v_y, a_x, a_y)$ at 25hz

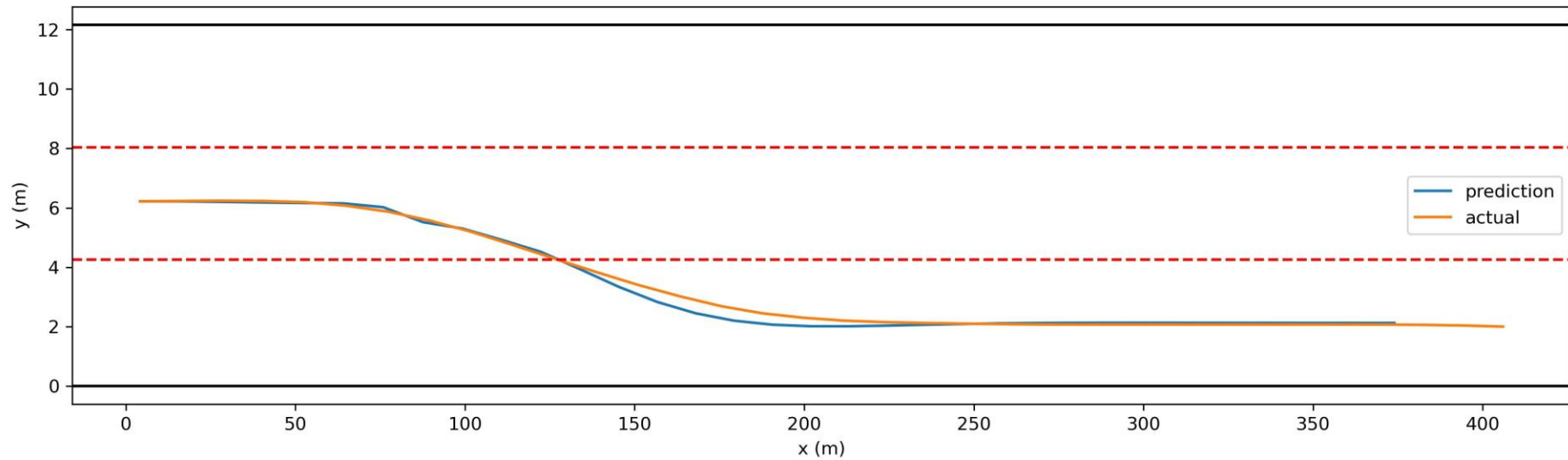
Input: consecutive states $[x(t), y(t), v(t), \text{psi}(t)]$ & $[x(t+1), y(t+1), v(t+1), \text{psi}(t+1)]$

Output: parameters of the cost function $[\eta, v_d, \alpha]$

Model Calibration

Optimization algorithm: differential evolution

Similarity measure: Fréchet distance



Trajectory 1422 from the HighD 50_tracks

Conclusion

Significance:

- We hope to understand driver preferences.
- The framework we propose improves existing microscopic models by taking into account interactions between vehicles.

Future study:

- We want to use the calibrated model as a testbed for designing optimal controller for AVs.

Thank you! Questions?



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